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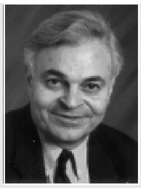

Haig Gulvanessian
Director, Construction Division, Building Research Establishment, Watford, UK;
Visiting Professor, Imperial College, London, UK

Milan Holicky
Deputy Director, Klokner Institute, Prague, Czech Republic;
Professor, Czech Technical University, Prague, Czech Republic

Eurocodes: using reliability analysis to combine action effects

H. Gulvanessian and M. Holicky

The head structural Eurocode for the design of buildings and other civil engineering works, *EN 1990 Basis of Structural Design*, establishes for all the Eurocodes the principles and requirements for safety, serviceability and durability. It provides alternative design procedures, for which national choice is allowed, in particular for the three fundamental combinations of action effects for the persistent and transient design situations in the verification of ultimate limit states. An example of a generic structural element shows that the alternative combinations may lead to considerably different reliability levels depending upon the ratio of variable actions to the total load. Probabilistic methods of structural reliability theory are used to identify characteristic features of each combination and to formulate general recommendations for the BSI National Annex for EN 1990.

NOTATION

E	action effect including model uncertainty
E_0	action effect without model uncertainty
E_d	design value of the action effect E
E_k	characteristic value of the action effect E
$F(\,)$	cumulative probability density function
$f(\,)$	probability density function
G	permanent action including model uncertainty, $G = \theta G_0$
G_0	permanent action without uncertainty
G_d	design value of the resistance G, $G_d = \gamma_G G_k$
G_k	characteristic value of the permanent action G
$g(\,)$	limit state function
k	ratio of variable action effects, $k = W_k/Q_k$
P	relevant representative value of prestressing action
p_f	failure probability
Q	main (dominant) variable action including model uncertainty, $Q = \theta Q_0$
Q_0	main (dominant) variable action without model uncertainty
Q_d	design value of the variable action Q, $Q_d = \gamma_Q Q_k$
Q_k	characteristic value of the variable action Q
R	resistance including model uncertainty
R_0	resistance without model uncertainty
R_d	design value of the resistance R, $R_d = \gamma_R R_k$
R_k	characteristic value of the resistance R
W	accompanying (non-dominant) variable action including model uncertainty, $W = \theta W_0$
W_0	accompanying (non-dominant) variable action without model uncertainty
W_d	design value of the variable action W, $W_d = \gamma_W W_k$
W_k	characteristic value of the variable action W
w_R	coefficient of variation of R
w_X	coefficient of variation of X
X	basic variables
α_R	coefficient of skewness of R
β	reliability index, $p_f = \Phi(-\beta)$
γ_G	partial factor for unfavourable permanent actions G
γ_M	partial factor for material property
γ_Q	partial factor for unfavourable variable actions Q
γ_R	partial factor for resistance R
γ_W	partial factor for unfavourable variable actions W
θ	model uncertainty
θ_R	coefficient of model uncertainty of R
θ_E	coefficient of model uncertainty of E
μ_R	mean of R
μ_X	the mean of X
ξ	reduction factor for unfavourable permanent actions G
σ_X	standard deviation of X
$\Phi(\,)$	distribution function of standardised normal distribution
χ	action effects ratio, $\chi = (Q_k + W_k)/(G_k + Q_k + W_k)$
$\psi_{0,i}$	reduction factor for combination value of load effect
ψ_Q	reduction factor for variable actions Q
ψ_W	reduction factor for variable actions W
ω	ratio of the mean to the characteristic value of R, $\omega = \mu_R/R_k$

1. INTRODUCTION

The complete suite of European Committee for Standardization (CEN) Structural Eurocodes, which presently exist in ENV (European pre-standard) form, will be converted to full EN (European normative standard) by 2004–2005. The first package of Eurocodes relating to the design of buildings should be converted earlier, by 2003–2004. The Eurocodes will be listed in the *Building Regulations Approved Document A*[1,2] as a way of meeting the requirements of Part A, and some of the requirements of Part B, of Schedule 1 of the Regulations.[1]

There are ten Eurocodes, each generally consisting of a number of parts, covering

- basis of structural design (EN 1990,[3] the head Eurocode)
- actions on structures (EN 1991)
- the design of the main structural materials (concrete, steel,

composite, timber, masonry and aluminium (EN 1992 to EN 1996 and EN 1999)
- geotechnical design (EN 1997)
- earthquake resistance (EN 1998).

Each Eurocode part will have a National Annex (which, for the UK, is published by the BSI), containing the Nationally Determined Parameters (NDPs) to be used for the structural design of buildings and civil engineering works in a Member State. Guidance Paper L[4] defines NDPs as 'a national choice left open in the EN Eurocodes about values (where symbols are given in the EN Eurocodes), classes or alternative procedures permitted within the EN Eurocodes'.

National Standards implementing a Eurocode part will comprise, without any alterations, the full text of the Eurocode and its annexes as published by CEN (see Fig. 1, blocks c, d and e. This is preceded by a National Title Page (a) and National Foreword (b) and will be followed by the National Annex (f).[5]

In the next few years, the structural Eurocodes will gradually be implemented in the Member States of CEN, together with their National Annexes, as National Standards.[5] Conflicting National Standards will be eventually withdrawn. This process will require important national decisions on the choice of NDPs that should be based on well-founded calibration. This paper:

- briefly describes the role of EN 1990 in the Eurocode system
- describes the investigation that provided the background information from which the choice of NDPs in EN 1990 relating to the alternative expressions for the combination of action effects, and the partial factors and coefficients (see section 2·1) within the expressions were made for the BSI National Annex to EN 1990
- describes the comparison of the various choices with current UK practice.

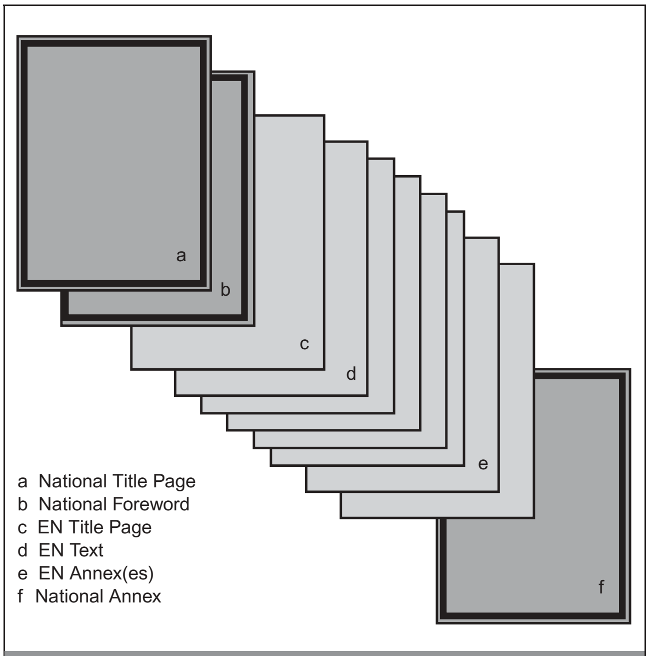

Fig. 1. National Standards implementing Eurocodes

The paper is based on a comprehensive investigation carried out for the Office of the Deputy Prime Minister (ODPM)/British Standards Institution (BSI) code consultancy scheme. The study was carried out primarily for buildings.

2. EUROCODE EN 1990: BASIS OF STRUCTURAL DESIGN

EN 1990 is the head code in the Eurocode suite and it establishes for all the Structural Eurocodes the principles and requirements for safety and serviceability. It also provides the basis and general principles for the structural design and verification of buildings and civil engineering structures (including geotechnical aspects) and gives guidelines for related aspects of structural reliability, durability and quality control. It is based on the limit state concept and is used in conjunction with the partial factor method.

As shown in Fig. 2, EN 1990 will be used with every Eurocode for the design of new structures, together with

- Eurocode 1: Actions on Structures, and
- the design Eurocodes (Eurocodes 2 to 9).

Thus the selected expressions for action effects from EN 1990 must be used for each of the design Eurocodes. A critical explanation of EN 1990 is given in Gulvanessian *et al.*[6]

2.1. Alternative load combination expressions in EN 1990

EN 1990 specifies three sets of alternative combination expressions for the determination of action effects

(i) expression (6·10) or
(ii) expressions (6·10a) and (6·10b) or
(iii) expressions (6·10a modified) and (6·10b)

(see below) for the persistent and transient design situations[3] to be used by EN 1991 and the design Eurocodes for ultimate limit state verification. Each case is now described in turn.

(i) The procedure using expression (6·10) is denoted as *Case A* in this paper.

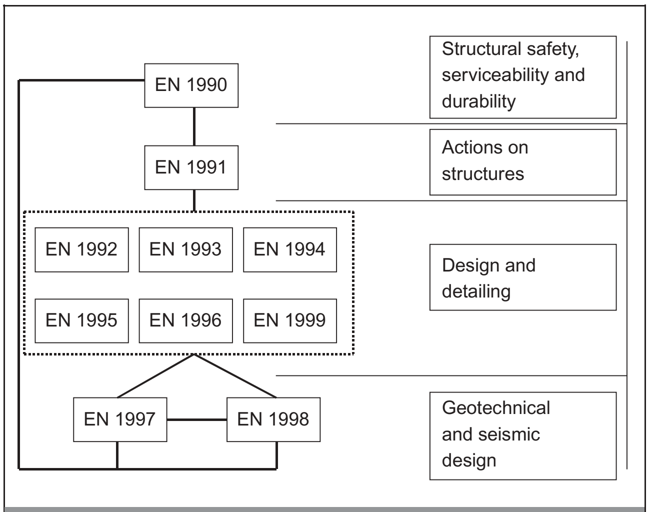

Fig. 2. Links between Eurocodes

$$\sum_{j\geqslant 1}\gamma_{G,j}G_{\mathrm{k},j}\ ''+''\ \gamma_P P\ ''+''\ \gamma_{Q,1}Q_{\mathrm{k},1}\ ''+''\ \sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{\mathrm{k},i} \qquad (6{\cdot}10)$$

In the above "+" means 'to be combined with', and Σ means 'the combined effect of'. For this case the combination of actions is governed by a leading variable action $Q_{\mathrm{k},1}$ represented by its characteristic value and multiplied by its appropriate safety factor γ_Q . Other variable actions $Q_{\mathrm{k},i}$ for $i >$ 1 which may act simultaneously with the leading variable action $Q_{\mathrm{k},1}$ are taken into account as *accompanying variable actions* and are represented by their combination value, i.e. their characteristic value reduced by the relevant combination factor ψ_0, and are multiplied by the appropriate safety factor to obtain the design values.

The permanent actions are taken into account with their characteristic values, and are multiplied by the load factor γ_G. Depending on whether the permanent actions act *favourably* or *unfavourably*, they produce different design values.[3]

(ii) The procedure using expressions (6·10a) and (6·10b) is denoted as *Case B* in this paper. The less favourable of the two following expressions is used

$$\sum_{j\geqslant 1}\gamma_{G,j}G_{\mathrm{k},j}\ ''+''\ \gamma_P P\ ''+''\ \gamma_{Q,1}\psi_{0,i}Q_{\mathrm{k},1}\ ''+''\ \sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{\mathrm{k},i} \qquad (6{\cdot}10\mathrm{a})$$

$$\sum_{j\geqslant 1}\xi\gamma_{G,j}G_{\mathrm{k},j}\ ''+''\ \gamma_P P\ ''+''\ \gamma_{Q,1}Q_{\mathrm{k},1}\ ''+''\ \sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{\mathrm{k},i} \qquad (6{\cdot}10\mathrm{b})$$

In expression (6·10a), there is no leading variable action: all the variable actions are taken into account with their combination value, i.e. their value is reduced by the relevant combination factor ψ_0. The *permanent actions* are taken into account as in expression (6·10), and the *unfavourable permanent actions* may be considered as the *leading action* in the combination of actions. All the actions are multiplied by the appropriate safety factors, γ_G or γ_Q.

In expression (6·10b) the combination of actions is governed by a *leading variable action* represented by its characteristic value as in expression (6·10) with the *other variable actions* being taken into account as *accompanying variable actions* and are represented by their combination value, i.e. their characteristic value is reduced by the appropriate combination coefficient of a variable action $\psi_{0,i}$. But the *unfavourable permanent* actions are taken into account with a characteristic value reduced by a reduction factor ξ, which may be considered as a combination factor.

All the actions are multiplied by the appropriate load factors γ_G or γ_Q. When the envelope of the two expressions showing the less favourable effects of expressions (6·10a) and (6·10b) is determined, generally expression (6·10a) applies to members where the ratio of variable action to total action is low and this is normally when a heavier structural material (e.g. concrete) is used, and (6·10b) applies where the same ratio is high and this is normally when a lighter structural material (e.g. steel) is used.

(iii) Expression (6·10a) above modified to include self-weight only and expression (6·10b). In this paper, this is referred to as *Case C*. This case is very similar to *Case B* but expression (6·10a modified) includes only permanent actions.

$$\sum_{j\geqslant 1}\gamma_{G,j}G_{\mathrm{k},j}\ ''+''\ \gamma_P P \qquad (6{\cdot}10\mathrm{a\ modified})$$

$$\sum_{j\geqslant 1}\xi\gamma_{G,j}G_{\mathrm{k},j}\ ''+''\ \gamma_P P\ ''+''\ \gamma_{Q,1}Q_{\mathrm{k},1}\ ''+''\ \sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{\mathrm{k},i} \qquad (6{\cdot}10\mathrm{b})$$

ξ is a reduction factor for unfavourable permanent actions G.

2.2. Choice of NDPs for the BSI National Annex to EN 1990

EN 1990[3] allows, through NDPs and the National Annexes, for national choice of

- which of the three combination expressions given in EN 1990[3] to use and
- appropriate safety factors (γ) and combination coefficients (ψ and ζ), for actions.

2.3. Partial factors and reduction coefficients in EN 1990

The partial factors and reduction coefficients γ, ψ and ξ recommended in EN 1990[3] and used in this investigation are summarised in Table 1.

2.4. Load combination expressions in BSI Codes of Practice

In this investigation a comparison is made with the combination expressions used in the BSI Codes of Practice. Although the expressions are in principle similar to expression (6·10) of EN 1990, the partial factors differ from those used in EN 1990 and given in Table 1. The γ factors specified in the BSI Codes of Practice and used in this investigation are given in Table 2.

3. INVESTIGATION OF THE COMBINATION EXPRESSIONS IN EN 1990

3.1. Objectives of the investigation

The principal objective of this investigation was to establish rules for the combination of action effects that would be common to all materials, and which of the three load

Action	Partial factors γ	Combination factor ψ	Reduction factor ξ
Permanent G	1·35	1·0	0·85
Imposed Q	1·5	0·7	—
Climatic W	1·5	0·6	—

Table 1. Partial and reduction factors (EN 1990)

	γ		
Action	Combination of *G* and *Q*	Combination of *G* and *W*	Combination of *G*, *Q* and *W*
Permanent *G*	1·4	1·4	1·2
Imposed *Q*	1·6	—	1·2
Climatic *W*	—	1·4	1·2

Table 2. Partial factors (BS 5628, BS 5950 and BS 8110)

combination formats given in EN 1990, together with partial and combination factors, is the most suitable for the UK National Annex for EN 1990 and in particular Annex A of EN 1990, Rules for Buildings. The choice was governed by the following considerations

- maintaining the reliability 'enjoyed' in the UK, given by the appropriate BSI Codes of Practice
- observing the Commission's recommendations,[7] including those with regard to NDPs deviating from recommended values
- the potential for achieving adequate consistency in reliability over the range of potential designs
- ease of use for designers, considering both the superstructure and the sub-structure
- the use of the same load combination rules and partial and combination factors for loading for all the materials
- economy.

3.2. Combination expressions

In investigating the three combination expressions given in section 2.1, the combination of three actions was considered: permanent action *G*, imposed load *Q* (assumed to be the leading variable action) acting in combination with wind action *W* (assumed to be the accompanying variable action). In the investigation described in this paper, linear structural behaviour and the same proportions between actions and action effects for all loads are assumed.

The design value of the action effect E_d is obtained using the characteristic values G_k, Q_k and W_k and appropriate partial factors γ_G, γ_Q, γ_W and reduction factors ξ, ψ_Q and ψ_W as follows for *Cases A*, *B* and *C*, described in section 2·1 (i), (ii) and (iii).

(i) *Case A (expression (6·10) of EN 1990)*. The design value of action effect E_d, assuming Q_k as the leading variable action and W_k as the accompanying variable action, is given as

$$E_d = \gamma_G G_k + \gamma_Q Q_k + \gamma_W \psi_W W_k \tag{1}$$

Equation (1) is similar in principle to the BSI expression for load combination, and the differences in the γ factors can be seen in Tables 1 and 2.

When one variable action is being considered in combination with the permanent action, for EN 1990:

$$E_d = 1{\cdot}35 G_k + 1{\cdot}5 Q_k \tag{2}$$

and for BSI

$$E_d = 1{\cdot}4 G_k + (1{\cdot}4 W_k \text{ or } 1{\cdot}6 Q_k) \tag{3}$$

When two variable actions are being considered (*Q* leading and *W* accompanying) in combination with the permanent action, then for EN 1990

$$E_d = 1{\cdot}35 G_k + 1{\cdot}5 Q_k + 0{\cdot}9 W_k \tag{4}$$

and for BSI

$$E_d = 1{\cdot}2 G_k + 1{\cdot}2 Q_k + 1{\cdot}2 W_k \tag{5}$$

(ii) Case B (twin expressions (6·10a) and (6·10b)).

$$E_d = 1{\cdot}35 G_k + 1{\cdot}5 \times 0{\cdot}7 Q_k + 1{\cdot}5 \times 0{\cdot}6 W_k \tag{6}$$

$$E_d = \xi 1{\cdot}35 G_k + 1{\cdot}5 Q_k + 1{\cdot}5 \times 0{\cdot}6 W_k \tag{7}$$

The less favourable action effect from equations (6) and (7) should be considered.

(iii) *Case C*. In addition, EN 1990[3] (through its Annex A) allows further modification of *Case B*. Modifying equation (6) by considering permanent loads only, the load effect is then

$$E_d = 1{\cdot}35 G_k \tag{8}$$

The less favourable action effect resulting from equations (7) and (8) is then considered.

Cases B and *C* do not have equivalent expressions in the BSI codes.

Should the *leading variable* action in equations (1) and (8) be the wind action *W*, the *now accompanying* imposed load *Q* will be reduced by the appropriate factor ψ_Q (note that in accordance with EN 1990, the partial factors for both variable actions are equal, $\gamma_Q = \gamma_W$).

3.3. Variation of resulting action effects with load ratio

The *resulting action effects* under various *intensities* of variable actions were investigated by using

- quantities χ given as the ratio of variable actions $Q_k + W_k$ to total load $G_k + Q_k + W_k$, and
- ratio *k* of accompanying action W_k to the leading action Q_k.

Thus

$$\chi = (Q_k+W_k)/(G_k+Q_k+W_k), \; k = W_k/Q_k \tag{9}$$

For the principal structural materials, a realistic range of χ is from 0·2 to 0·6. However in some cases the load ratio χ may be very low if not zero (e.g. roof of underground garage loaded by its own weight and earth weight).

For *Case A*, equation (1) is valid in the whole range $0 \leqslant \chi \leqslant 1$. For *Case B*, equation (6) is valid in the range $0 \leqslant \chi \leqslant \chi_{\text{lim,B}}$ and expression (7) in the range $\chi_{\text{lim,B}} \leqslant \chi \leqslant 1$. Correspondingly, for *Case C* equation (8) is valid in the range $0 \leqslant \chi \leqslant \chi_{\text{lim,C}}$ and equation (7) in the range $\chi_{\text{lim,C}} \leqslant \chi \leqslant 1$. The limiting values for χ are illustrated in Figs. 5, 7 and 8, later. The limiting values $\chi_{\text{lim,B}}$ and $\chi_{\text{lim,C}}$ are derived from equations (6)–(9) and are given in the literature.[8]

4. GENERIC STRUCTURAL MEMBER

To investigate the relative levels of safety for *Cases A*, *B and C*, a resistance model for a generic structural member, defined below, was assumed.

For the generic structural member used in this investigation it is assumed that the characteristic value R_k of the resistance R may be defined as the 5% fractile of R and the design value of the resistance R_d as

$$R_d = R_k/\gamma_M \tag{10}$$

where γ_M denotes the partial factor for the material property, also accounting for model uncertainties and dimensional variations.

The significance of both values R_k and R_d is illustrated in Fig. 3. The random variable R is described by the probability density function $\Phi_R\,(R)$, and the design value R_d is indicated as a particular value of R corresponding to a certain small probability p of being violated.

Table 3 shows the assumed values for the partial factor of the material property γ_M and the coefficient of variation w_R, used in the reliability analysis in this investigation for a generic structural member, for the Eurocodes and BSI codes. BSI codes use lower values of γ_M than Eurocodes. However, in BSI codes γ_M is generally applied to values that are generally not the 5% characteristic value. The factor $\gamma_M = 1{\cdot}10$ was selected as a middle value applied to the 5% characteristic value in order to enable direct comparisons with the Eurocodes.

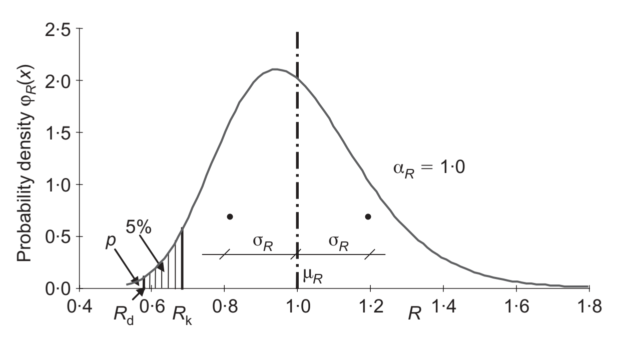

Fig. 3. Random variable R, the characteristic value R_k and design value R_d

	Typical value	Range
Eurocodes, $\gamma_M = R_k/R_d$	1·15	1·0–1·3
BSI codes, $\gamma_M = R_k/R_d$	1·10	1·0–1·20
The coefficient of variation w_R	0·15	0·10–0·25
The mean factor $\omega = \mu_R/R_k$	1·28	1·10–1·40

Note: The coefficient of variation w_R includes the variability of the model uncertainty assumed to have the coefficient of variability 0·05

Table 3. Global resistance factor γ_M, the coefficient of variation w_R and the mean factor ω

5. PRINCIPLES OF RELIABILITY ANALYSIS

5.1. Limit state function

The key step in the reliability analysis for this investigation concerns the definition of a limit state function (reliability margin) g(X) separating the safe and unsafe domain of the basic variables X. In this study the limit state function g(X) is the difference between the resistance $R(X)$ and the action effect $E(X)$

$$\text{g}(X) = R(X) - E(X) = \theta_R R_0(X) - \theta_E E_0(X) \tag{11}$$

where θ_R represents uncertainties of the resistance model $R_0(X)$ and θ_E represents uncertainties of the action effect model $E_0(X)$. Thus if the resistance $R(X)$ is greater than the load effect $E(X)$, then the structure is safe; if the resistance $R(X)$ is less than the load effect $E(X)$, the structure is unsafe. It thus follows from equation (11) that the safe domain is described by the inequality $g(X) > 0$, the unsafe domain by $g(X) < 0$. The boundary separating the safe and unsafe domain is given by $g(X) = 0$.

5.1.1. Example. A steel tie structural member having a resistance $R = Af_y$, where A denotes the cross-section of the member and f_y the yield point is loaded by an axial tensile force $E = Q$. Assuming that there are no model uncertainties ($\theta_R = \theta_R = 1$) the limit state function (11) becomes

$$\text{g}(X) = Af_y - Q \tag{12}$$

In this case the basic variables X consist of three variables A, f_y and Q.

Taking into account the general equations (1)–(8), the load effect $E(X)$ including load uncertainty may be written as

$$E(X) = \theta_E(G_0 + Q_0 + W_0) \tag{13}$$

It follows that the basic variables R, G, Q and W covering the effects of model uncertainties are defined as:

$$\begin{aligned} R &= \theta_R R_0(X) \\ G &= \theta_E G_0 \\ Q &= \theta_E Q_0 \\ W &= \theta_E W_0 \end{aligned} \tag{14}$$

Thus, considering equations (14), the limit state function, in equation (11), may be written in a simplified form as

$$g(X) = R - (G + Q + W) \tag{15}$$

Note that the cumulative basic variables R, G, Q and W in equation (15) include the effects of the factors of model uncertainties θ_R and θ_E (see equation (14)).

5.2. Probabilistic models of basic variables

In this investigation it is assumed that the structural members are designed economically, so that the design value of the resistance $R_d(X)$ equals the design value of the load effect $E_d(X)$

$$R_d(X) = E_d(X) \tag{16}$$

In normal practice the design value of the resistance $R_d(X)$ is greater than the design load effect $E_d(X)$, which provides an additional safety margin, not considered here.

Assuming a certain set of partial and combination factors γ, ψ and ξ, the design equation (16) can be used to specify the characteristic values X_k of each basic variable X. The probabilistic characteristics (mean, standard deviation) of each basic variable X can be then related to its characteristic value X_k as indicated in Table 4.

The probabilistic models indicated in Table 4 are based on data available in the literature,[9] recommendation by JCSS[10] and the experience of the authors. As mentioned above, the probabilistic characteristics indicated in Table 4 represent just conventional models that might be slightly conservative. It should be also noted that the models indicated in Table 4 might not be universally applicable to all types of imposed loads.

Note that the mean of a resistance R indicated in Table 4 in terms of the characteristic value R_k and the standard deviation σ_R may be assessed assuming a given coefficient of variation w_R using the relationship

$$\mu_R = R_k \exp(1{\cdot}65 w_R) \tag{17}$$

Under this assumption the mean resistance factor ω considered in Table 4 is given as

$$\omega = \mu_R / R_k = \exp(1{\cdot}65 w_R) \tag{18}$$

Considering the coefficient of variation $w_R = 0{\cdot}15$, the mean resistance factor becomes $\omega = 1{\cdot}28$.

It should be emphasised that the probabilistic models of basic variables indicated in Table 4 are primarily intended as 'conventional models' in time-invariant reliability analysis of generic structural members using Turkstra's combination rule[11] (explained also in ISO 2394[12]) for the probabilistic calibration of the rules for combinations of actions.

The conventional models indicated in Table 4 should enable objective comparison of results of various reliability studies expected in the near future in connection with the implementation of the present suite of Eurocodes into the national systems of design codes. However, when the reliability of different types of structural members under particular conditions is assessed, the proposed models in Table 4 may have to be adjusted to the specific conditions of the analysed structural member.

5.3. Reliability measures

The probability of failure p_f is the basic reliability measure used in this study. It can be expressed on the basis of a limit state (performance) function $g(X)$ defined in such a way that a structure is considered to survive if $g(X) > 0$ and to fail if $g(X) \leqslant 0$. An example of the function $g(X)$ is given by equation (15). In a general case the failure probability p_f can be determined using the integral

No.	Category of variables	Name of basic variables	Sym. X	Distribution†	Mean μ_X	St. dev. σ_X
1	Actions	Permanent	G_0	N	G_k	$0{\cdot}1\mu_X$
2		Imposed, 5 years	Q_0	GU	$0{\cdot}2Q_k$	$1{\cdot}1\mu_X$
2		Imposed, 50 years	Q_0	GU	$0{\cdot}6Q_k$	$0{\cdot}35\mu_X$
3		Wind, 1 year	W_0	GU	$0{\cdot}3W_k$	$0{\cdot}5\mu_X$
4		Wind, 50 years	W_0	GU	$0{\cdot}7W_k$	$0{\cdot}35\mu_X$
5	Resistance	Resistance	R	LN	$R_k + 1{\cdot}65\sigma_R$	$0{\cdot}15\mu_X$
6	Uncertainty	Uncertainty	θ_E	LN	1	0·05

† N, normal distribution; GU, Gumbell distribution; LN, log-normal distribution

Table 4. Probabilistic models of basic variables for time invariant reliability analysis using Turkstra's rule. (In accordance with Turkstra's rule the probabilistic models corresponding to a 50-year return period are used for leading variable actions and models corresponding to annual maximum or 5-year maximum for accompanying actions.)

$$p_\mathrm{f} = \mathrm{prob}(g(X) \leqslant 0) = \int_{g(X)\leqslant 0} f_g(X)\,\mathrm{d}X \tag{19}$$

where $f_g(X)$ denotes the joint probability density distribution of the basic variables X, which may have to be assumed on the basis of experience and judgement.

Assume that both the resistance $R(X)$ and the load effect $E(X)$ represent a single variable Z used to analyse structural performance (e.g. axial force or bending moment that is represented by $R(X)$ and $E(X)$). Then the integration indicated in equation (19) may be simplified and the probability p_f can be expressed as

$$p_\mathrm{f} = \mathrm{prob}(g(X) \leqslant 0) = \int_{-\infty}^{\infty} f_E(Z) F_R(Z)\,\mathrm{d}Z \tag{20}$$

where $f_E(Z)$ denotes the probability density function of $E(X)$ and $F_R(Z)$ the cumulative distribution function of $R(X)$. This is shown diagrammatically in Fig. 4.

To use expression (20) both the probability density function $f_E(Z)$ and the distribution function $F_R(Z)$ must be known (at least in an approximate form). A simplified procedure based on expression (20) was used in this investigation.

In Annex C of EN 1990 an alternative measure of reliability is conventionally defined by the reliability index β, which is related to the probability of failure p_f as

$$p_\mathrm{f} = \Phi(-\beta) \tag{21}$$

where Φ is the cumulative distribution function of the standardised normal distribution. The relation between p_f and β is indicated in Table 5.

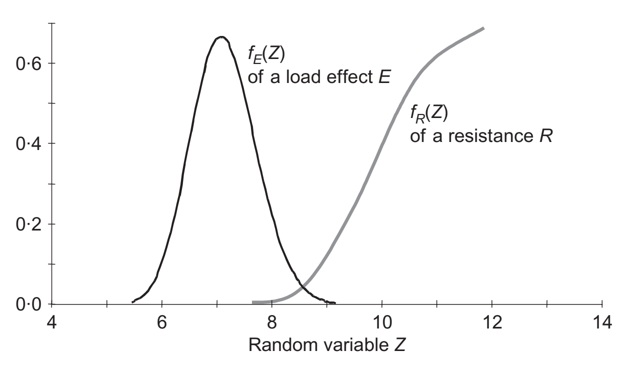

Fig. 4. Probability density $f_E(Z)$ and cumulative distribution function $F_R(Z)$

p_f	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
β	1·28	2·32	3·09	3·72	4·27	4·75	5·20

Table 5. Relation between β and p_f

Table C2 of EN 1990[3] recommends for the ultimate limit state of buildings over a 50-year design working life a minimum target value of reliability index $\beta_\mathrm{t} = 3{\cdot}8$, which is equivalent to $\beta_\mathrm{t} = 4{\cdot}7$ for a one-year design life. Both the equivalent reliability measures, the failure probability p_f and the reliability index β, are used in this study.

6. RESULTS FROM THE EVALUATION

6.1. General comments on the evaluation

Selected results of the reliability analysis from the investigation are presented in graphical form that indicates the variation of the reliability index β with the load ratio χ.

It should be appreciated that the results generally obtained from a reliability analysis prove very useful for relative comparisons, and only in detailed very comprehensive investigations provide absolute results. However, considering that the input parameters and distributions used for the 'basic variables' are those recommended by the JCSS, the results obtained can be considered as generally sound.

6.2. Combination of permanent action with one variable action

For this analysis the imposed load Q acts alone with the permanent actions G (i.e. $k = 0$ in equation (9)). G and Q have the probabilistic model characteristics given in Table 4. The EN 1990 recommended values given in Table 1 were used, as were those given in Table 2 for the BSI codes. The middle values from Table 3 were used for the global resistance factor γ_M and the coefficients of variation w_R (i.e. $\gamma_\mathrm{M} = 1{\cdot}15$ for EN 1990 and $\gamma_\mathrm{M} = 1{\cdot}10$ for BSI codes and $w_R = 0{\cdot}15$).

Figure 5 shows a comparison of the EN 1990 combination rules, *Case A* (i.e. expression (6·10)), *Case B* (i.e. expressions (6·10a) and (6·10b)) and *Case C* (i.e. expressions (6·10a modified) and (6·10b)), with the BSI rules (i.e. BS 8110, BS 5950, etc.). (Note in Figs. 5, 7 and 8 *Case A*, *Case B* and *Case C* are denoted A, B and C, respectively.)

It follows from Fig. 5 that for the assumed coefficient of variation $w_R = 0{\cdot}15$, the combination rule *Case A* (i.e. expression (6·10) of EN 1990) seems to be fully acceptable ($\beta > 3{\cdot}8$ and $p_\mathrm{f} < 7{\cdot}23 \times 10^{-5}$) in the interval $0 < \chi < 0{\cdot}8$. However, the reliability level varies considerably with χ, indicating possible uneconomic designs for $0{\cdot}2 < \chi < 0{\cdot}5$. *Case A* does however provide a safety level very similar to that obtained from BSI codes.

Case B (i.e. expressions (6·10a) and (6·10b) of EN 1990) is acceptable in a slightly shorter range of χ ($0 < \chi < 0{\cdot}7$) than *Case A* and that obtained from BSI codes, but provides a much more uniform distribution of reliability level with χ. Obviously, *Case B* leads to a more economic design than *Case A* and BSI codes.

Case C (i.e. expressions (6·10a modified) and (6·10b) of EN 1990) gives lower reliability levels particularly for the interval $0 < \chi < 0{\cdot}3$ and the authors do not recommend its

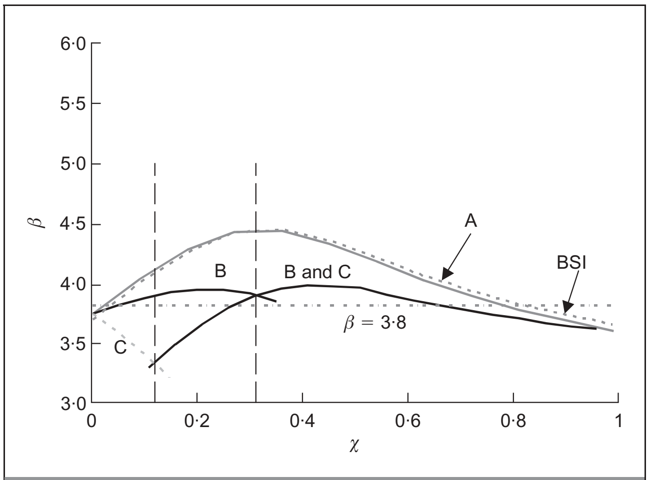

Fig. 5. Variation of reliability index β with χ for EN 1990 combination rules *Cases A, B* and *C*, ($\gamma_R = 1·15$, $\gamma_G = 1·35$, $\gamma_Q = 1·5$), and BSI rules ($\gamma_R = 1·10$, $\gamma_G = 1·4$, $\gamma_Q = 1·6$), with $k = 0$

use unless the partial factors γ are increased.

Similar results were obtained for structural members made of specific materials (e.g. concrete, steel or timber).

Figure 6 shows the variation of the reliability index β with the load ratio χ for the partial factor for resistance γ_R between 1·0 and 1·5 for *Case A*. It follows from Fig. 6 that for the assumed variables the *acceptable* domain of the load ratio χ and the coefficient of variation w_R is limited by the contour line determined as an intersection of the β surface and the plane $\beta = 3·8$ in Fig. 6. Obviously, with increasing γ_R, the reliability index β increases. Note that the reliability level corresponding to $\gamma_R = 1·0$ is entirely below the level $\beta = 3·8$. To achieve a satisfactory reliability level $\beta > 3·8$ in a realistic range of the load ratio $0·2 < \chi < 0·6$, it would be necessary to use a material factor $\gamma_R = 1·10$ or greater.

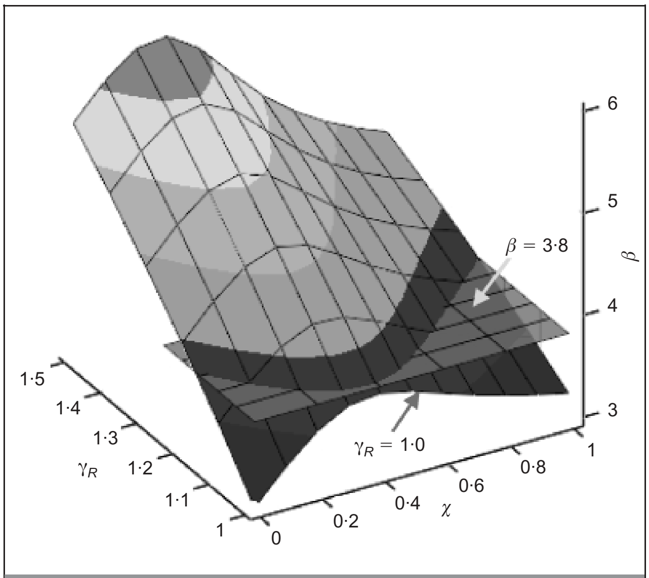

Fig. 6. Variation of the reliability index β with the load ratio χ and partial factor for resistance γ_R for values between 1·00 and 1·5 for $k = 0$ (i.e. imposed load Q acting alone) for *Case A* (expression (6·10)) and for $\psi_G = 1·35$, $\psi_Q = 1·5$ and $w_R = 0·15$

6.3. Combination of permanent action with two variable actions

For this analysis the imposed load Q acts with an accompanying action the action W with the permanent action G (it is assumed that $k = 0·25$ in equation (9)). G, Q and W have the probabilistic model characteristics given in Table 4. The EN 1990 recommended values given in Table 1 were used, as were those given in Table 2 for the BSI codes. The middle values from Table 3 were used for the global resistance factor γ_M and the coefficients of variation w_R (i.e. $\gamma_M = 1·15$ for EN 1990 and $\gamma_M = 1·10$ for BSI codes and $w_R = 0·15$).

Figure 7 shows a comparison of the EN 1990 combination rules for *Case A*, *Case B* and *Case C* with the BSI rules. It follows from Fig. 7 that, for the assumed coefficient of variation $w_R = 0·15$, the reliability of the generic structural member exposed to two variable actions is considerably greater than the same cross-section exposed to one variable action (Fig. 5). In addition, the levels of safety for *Case A* and *Case B* are considerably greater than that obtained for BSI codes. Both *Cases A* and *B* seem to be fully acceptable for the whole range of χ. The levels of reliability obtained for the BSI codes show the target of reliability index $\beta < 3·8$ over the whole range of χ.

Similar results were obtained for structural members made of specific materials (e.g. concrete, steel or timber).

6.4. Effects of varying ζ for Case B (expressions (6.10a) and (6.10b) of EN 1990)

The effect on β on varying ζ from 0·85 (the EN 1990 recommended value) to 0·90 and 0·95 is shown in Fig. 8.

7. OBSERVATIONS

1. When one variable action is considered in combination with the permanent actions, the adoption of combination *Case A* (i.e. expression (6·10) of EN 1990) using the EN 1990 recommended values for partial safety and combination factors will produce a closely comparable reliability to that ‘enjoyed’ in the UK (see Figs. 5 and 6).

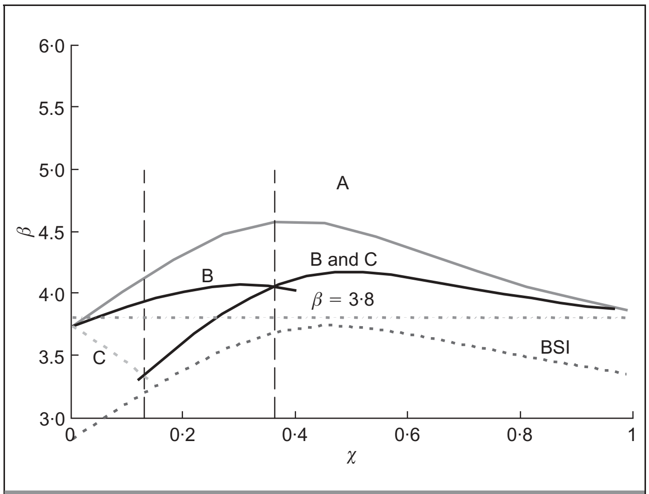

Fig. 7. Variation of reliability index β with χ for the EN combination rules *Cases A, B* and *C*, ($\gamma_R = 1·15$, $\gamma_G = 1·35$, $\gamma_Q = 1·5$, $\gamma_W = 1·5$, $\psi_W = 0·6$) and the BSI rules ($\gamma_R = 1·10$, $\gamma_G = 1·2$, $\gamma_Q = 1·2$, $\gamma_W = 1·2$), for $k = 0·25$

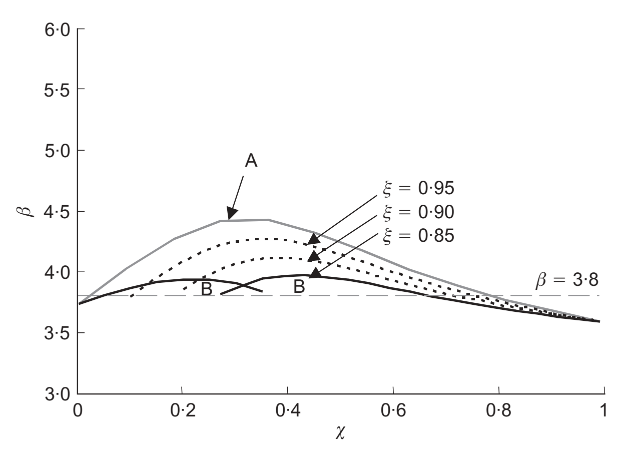

Fig. 8. Variation of reliability index β with χ for EN 1990 combination rules *Case A* ($\gamma_R = 1{\cdot}15$, $\gamma_G = 1{\cdot}35$, $\gamma_Q = 1{\cdot}5$) with $k = 0$ and *Case B* ($\gamma_R = 1{\cdot}15$, $\gamma_G = 1{\cdot}35$, $\gamma_Q = 1{\cdot}5$, $\xi = 0{\cdot}85, 0{\cdot}90, 0{\cdot}95$) with $k = 0{\cdot}25$

2. The reliability levels when considering two variable actions acting in combination with the permanent actions is much higher for EN 1990, *Cases A*, *B*, and *C* than that obtained by the BSI codes. The BSI recommendation of using identical partial factors ($\gamma = 1{\cdot}2$) for the permanent action and for each of the variable actions has been shown to give a lower level of reliability than $\beta = 3{\cdot}8$ (see Fig. 7).
3. Figs. 5 and 7 indicate that *Case A* (expression (6·10)) does not produce a consistent level of safety for the complete range of χ. Adopting *Case B* (expressions (6·10a) and (6·10b)) provides a more consistent level, but a lower level of reliability than that currently 'enjoyed' in the UK.
4. Table 6 discusses the attributes of using *Cases A*, *B* and *C* with recommended γ, ψ and ζ values from EN 1990 and their effects on the objectives outlined in section 3·1.
5. Fig. 8 shows the variation of β for the complete range of χ for
   - *Case A*, with $\gamma_G = 1{\cdot}35$, $\gamma_Q = 1{\cdot}5$
   - *Case B*, with $\gamma_G = 1{\cdot}35$ $\gamma_Q = 1{\cdot}5$, $\xi = 0{\cdot}85$ $\psi_Q = 0{\cdot}7$, $\psi_W = 0{\cdot}6$
   - *Case B*, with $\gamma_G = 1{\cdot}35$ $\gamma_Q = 1{\cdot}5$, $\xi = 0{\cdot}90$, $\psi_Q = 0{\cdot}7$, $\psi_W = 0{\cdot}6$
   - *Case B*, with $\gamma_G = 1{\cdot}35$ $\gamma_Q = 1{\cdot}5$, $\xi = 0{\cdot}95$, $\psi_Q = 0{\cdot}7$, $\psi_W = 0{\cdot}6$

   Comparing the three curves for *Case B* with *Case A* (with $\gamma_G = 1{\cdot}35$ and $\gamma_Q = 1{\cdot}5$), noting that *Case A* also agrees with BSI shows that use of *Case B* with $\xi = 0{\cdot}925$ (note that reduced $\gamma_G = 0{\cdot}925 \times 1{\cdot}35 = 1{\cdot}25$) instead of 0·85 (note that reduced $\gamma_G = 0{\cdot}85 \times 1{\cdot}35 = 1{\cdot}15$) will provide a lower reduction of β of about 5% between χ of 0·2 and 0·5, and better consistency of reliability compared to *Case A*.

8. CONCLUSIONS AND RECOMMENDATIONS FOR COMBINATION AND PARTIAL FACTORS TO BE ADOPTED IN THE BSI NATIONAL ANNEX FOR EN 1990

Based on the discussion in sections 6 and 7, the following two combinations have been adopted in the BSI National Annex for EN 1990.

1. Expression (6·10) with $\gamma_G = 1{\cdot}35$ and $\gamma_Q = 1{\cdot}5$.
2. Expressions (6·10a) and (6·10b) with $\gamma_G = 1{\cdot}35$, $\gamma_Q = 1{\cdot}5$, $\xi = 0{\cdot}925$, $\psi_Q = 0{\cdot}7$, $\psi_W = 0{\cdot}6$.

8.1. Adoption in the National Annex of expression (6.10) with $\gamma_G = 1.35$ and $\gamma_Q = 1.5$

Adopting this formulation has the following *advantages*

- the BSI National Annex will be specifying the EN 1990 recommended safety factors γ_G and γ_Q
- it provides the level of safety currently 'enjoyed' in the UK when considering one variable action, and a greater level of safety when considering more than one variable action in the combination of action effects
- as for existing UK practice, adopting this recommendation

Objective (see section 3.1)	*Case A* (6·10) of EN 1990	*Case B* (6·10a) and (6·10b) of EN 1990	*Case C* (6·10a modified) and (6·10b) of EN 1990
Same level of reliability as BSI codes? (as measured by index β)	Yes	No. 10–15% lower for χ between 0·15 and 0·6	No. 10–20% lower for χ between 0·1 and 0·6
Consistency of reliability for range of χ	No. Higher reliability for χ between 0·2 and 0·6	Yes	No. Lower reliability for $\chi < 0{\cdot}3$
Usability	As for BSI codes	More complicated than BSI codes. Studies show *Case B* can be very complicated for special considerations. Problems envisaged for sub-structures	Slightly more complicated than BSI codes. Problems envisaged for sub-structures
Economy Considering action effects *only* for a given resistance	As for UK practice	Greater economy for χ between 0·15 and 0·6	Much greater economy for χ between 0·15 and 0·6

Notes: whilst β may be only '10–15% lower', this reflects a 40-fold increase in the probability of failure
Special attention (e.g. increased load factors) should be paid when $\chi > 0{\cdot}6$, and also for the BSI expressions

Table 6. Comparison of *Cases A*, *B* and *C* with BSI codes and UK practice

would deliver, for χ between 0·2 and 0·6, a reliability index β of approximately 4·3, rather than the minimum target of 3·8. Annex B of EN 1990 would relate this higher level of target to situations with 'large' expected consequences of failure CC3 and 'moderate' relative cost of safety measures or RC3 overall
- the same usability as BSI codes.

Adopting this formulation has the following *disadvantage*

- poor consistency of safety over the whole range of χ.

8.2. Adoption of expressions (6.10a) and (6.10b) with $\gamma_G = 1.35$ and $\gamma_Q = 1.5$ and $\xi = 0.925$

Adopting this formulation has the following *advantages*

- the BSI National Annex will be specifying the EN 1990 recommended safety factor γ_G and γ_Q
- if this formulation is specified *in addition* to expression (6·10) with $\gamma_G = 1{\cdot}35$ and $\gamma_Q = 1{\cdot}5$, then the BSI National Annex will be providing a tool for obtaining a greater consistency of safety over the complete range of χ.

Adopting this formulation has the following *disadvantage*

- if this formulation is specified *in place* of expression (6·10), problems arise with usability.

9. CONCLUDING REMARKS

The newly available EN 1990 provides alternative design procedures that lead, in some cases, to significantly different reliability levels. The Eurocodes recognise the responsibility of the Regulatory and other National Competent Authorities in each Member State and have safeguarded their right to determine values related to safety matters at national level. Thus the determination of NDPs relating to safety levels for the National Annexes is an important task for each Member State.

Simple examples for a generic structural member show that designs according to the alternative combination rules provided in EN 1990 by expressions (6·10) and (6·10a) and (6·10b) may vary considerably. Expression (6·10) leads to the most reliable (but in some cases to uneconomical) structures but provides the same level of reliability currently enjoyed in the UK. Expressions (6·10a) and (6·10b) provide a lower but comparatively most uniform reliability level for all load ratios. Moreover, these expressions together with recommended partial factors seem to comply fully with EN recommendations (reliability index 3·8 for a 50-year time period). The lowest reliability is obtained from the third alternative, given by the modified expression (6·10a) and expression (6·10b). This alternative seems to lead to a rather low reliability level, particularly for structures exposed mainly to a permanent load; it is not recommended for use.

10. ACKNOWLEDGEMENTS

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