A FULLY COUPLED THERMO-HYDRO MECHANICAL ANALYSIS OF THE IMPACT OF TEMPERATURE AND HUMIDITY VARIATION ON THE STATE OF HISTORICAL STONE BUILDINGS

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ABSTRACT

In order to understand, explain, predict and prevent the degradation of historical monuments in the valley of Loire in the region of Centre (in France) the authors of this paper have worked in the framework of a regional project baptized SACRE (http://www.crmd-sacre.com/) in the modeling of THM phenomena that are susceptible to take place on the walls of historical buildings and play a major role on the long-term degradation process. As part of this research program a numerical modeling of daily and annually variation on temperature and humidity was performed. The objectives of such a modeling are multiplies: firstly from an inverse analysis on measured data on temperature and humidity variation in various depth of a stone wall in Chambord Castle we try to asses the in situ values of some key parameters that control the humidity transport in the stone pores such as relative permeability, effective Fick’s coefficient, thermal expansion coefficient of stones, etc. Secondly, once the full picture of the phenomena known and the parameters adjusted, some simulations are performed with the objective to produce at least some of most typical degradation forms observed in situ. This paper describes the numerical simulations performed in the framework of the first objective. The results show that a non-uniform gradient of temperature and humidity on the stone walls is a result of daily meteorological changes and could leads to differential stress inside stones and on interfaces with stones and mortars.

INTRODUCTION

The preservation of historical and cultural monuments from weathering is a challenge topic not only from cultural heritage and political point of view, but also from scientific one. This evidence has resulted in a number of initiatives from european, nationals and local authorities to encourage the scientific research to understand the mechanisms of weathering and to establish strategies of preservation and/or restoration of stone buildings. The research program SACRE (http://www.crmd-sacre.com/) is a local financed program from Center Region (FRANCE) dedicated to the diagnostic, understanding and prevision of weathering of Chambord castle, one of the most known from Loire castles. Various weathering has been observed in different parts of the castle, even though the castle is situated in a natural protected site and no particular urban pollution form the immediate vicinity is assumed. In order to explain weatherings in such conditions different hypothesis are formulated such as ([2]): dissolution of minerals/salt from some parts of stones in the saturating fluids and their precipitation in pores; crack initiation and growth due to the variation of climate conditions; thermo-hydro-mechanical incompatibility of components; biological activity and so on. The role of the meteorological conditions in the weathering of stone buildings, and particularly on that of Chambord castle, is one of objectives of SACRE research program. For that purpose a two-ways investigation of climate in the surrounding of Chambord

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castle has been performed: On one hand data are collected from archives of meteorological stations of Meteo France in the close surrounding of Chambord castle in order to reconstitute the conditions of temperature and humidity evolution. On the other hand measurements of temperature and relative humidity (RH) are performed in continue on the air and inside some hollow boreholes into the walls of Chambord castles at 15mm, 30mm, 50mm and 250 mm. These measures aim at quantitative evaluation of the role of climate conditions on stone weathering as a first step towards a more complex physico-chimical evaluation of environmental impact on stone weathering.

This paper deals with hydromechanical behavior modeling of a stone wall, typical a Chambord stone-wall, under real meteorological data. While the problem has been often cited as a possible factor on weathering, and different authors have attempted to give a solution ([3], [4]), in this work an attempt is made to correlate on hand the measured data from meteorological station and in-borehole measured data through a hydro-mechanical model, and on the other hand these variations of temperature and relative humidity with stress variation and so with a potential initiation and growth of cracks. We begin with a short description with the model used in calculations and back analyze of in situ measurement. Then a thermo-hydro-mechanical (THM) modeling of a typical wall from Chambord is analyzed in order to quantify the impact of temperature and relative humidity in internal stress distribution. In all this paper an underlined symbol ($\underline{\alpha}$) denotes a vector, a double underlined symbol ($\underline{\underline{\alpha}}$) indicates a second order tensor.

A THERMO-HYDRO-MECHANICAL MODEL FOR STONES

Stones of buildings are considered as soft rocks or hard soils and models developed for geomaterials are believed to fit good to behavior of stones ([3], [4],[5]). So it is so quite normal to find that the same experimental and numerical tools used to geomaterials are adopted with minor modifications to stones. From this point of view this work follows this same trend attempting to apply the theory of unsaturated porous media to thermo-hydro-mechanical behavior of tuffeau, the stone used for construction of Chambord castle. We believe that this same approach could be used for other stones and other conditions with minor modifications.

In this modeling approach stone is considered as an unsaturated porous media constituted of a solid phase and of a system of pores connected or closed. This pores system contains of the liquid phase comprising only pure water and a gaseous phase constituting of dry air and water vapor (the chemical aspects are neglected for the moment). The formulation of the model is based on the theory of Mechanics of Porous Media ([1]) while the analyses are performed by using Code_Aster, an open source finite element code developed by EDF-FRANCE ([5], http://www.code-aster.org) A detailed description of the full coupling model used in these analyses is beyond the scope of this paper (see for example [1] and [5] for more of details) but essentials equations of non linear thermo-hydro-mechanical behavior and coupling are resumed.

The key assumption of the model is the existence for the unsaturated porous media, of an effective stress taking into account the contribution on the mechanical behavior of the temperature and relative humidity:

$$\sigma^\prime = \sigma + b\pi + 3\alpha K \Delta T \delta$$

The left hand quantity in eq.1 is the effective stress tensor which counts for mechanical loading (stress tensor $\sigma$) as well as hydraulic and thermal ones. In fact the second and third terms in Eq. (1) present coupling terms respectively for hydro mechanical and thermo mechanical behavior through Biot’s coefficient ($b = 1 - K / K_s$) and the thermal expansion coefficient of skeleton $\alpha$. Two
parameters $K$ and $K_s$ are bulk modulus of the drained medium and solid grains, respectively, while $\mathbf{g}$ is a unit second order tensor. The symbol $\pi$ has the dimension of pressure which is exactly interstitial pressure in the case of saturated media. Its expression for unsaturated media is written as function of liquid saturation $S_i$ and capillary pressure $P_c$ (difference of the gaseous and liquid pressure $P_c = P_g - P_l$) ([1]):

$$\pi = \begin{cases} 
S_i(P_c)dP_c & \text{if } S_i < 1 \\
1 & \text{if } S_i > 1 
\end{cases}$$

(2)

In the framework of the problem treated here the pressure of gaseous phase is supposed to be constant and equal to atmospheric pressure, and so Richard’s hypothesis is supposed valuable. The function $S_i(P_c)$ under the integration symbol represents the isothermal sorption curve of material which must be evaluated by laboratory tests. It is found that this function could be reasonably approximated by an empirical expression due to Van Genuchten [7]:

$$S_i(P_c) = \left[1 + \left(\frac{P_c}{P_r}\right)^n\right]^{\frac{1-n}{n}}$$

(3)

In this equation $P_r$ (stress units) and $n$ (dimensionless) are two fitting parameters. The advection of both gaseous and liquid phase is given by a generalized Darcy’s law making use of so effective permeability:

$$\frac{M_j}{\rho_j} = -\lambda_j \nabla (P_j)$$

(4)

In this equation $M_j$ is the flux vector and $\lambda_j$ is the effective hydraulic conductivity in unsaturated media of $j^{th}$ phase. The effective hydraulic conductivity in unsaturated media of $j^{th}$ phase is defined as it follows:

$$\lambda_j = \frac{k_{in}k_{rel}^j(S_j)}{\mu_j}$$

(5)

with $k_{in}$ being the intrinsic permeability and $k_{rel}^j(S_j)$ the relative permeability, a limited value function of liquid saturation $S_j$ taking values between 0 and 1. In present model the relative permeability of liquid phase is considered proportional to liquid saturation ($\sim S_j^3$) while that of gaseous phase is relative permeability of gases is proportional to $(1 - S_j^3)$. Furthermore, the transfer in this later phase is completed by the Fick’s diffusion which represents the relative movement of the vapor in the mixture of gases:

$$\frac{M_v}{\rho_v} = -F \nabla (C_v)$$

(6)

The parameter $F$ is the Fick’s coefficient and $C_v = P_v/P_g$ is the concentration of vapor in gaseous phase. This equation can be rewritten as follow by considering of the uniform pressure of gases:

$$\frac{M_v}{\rho_v} = -\frac{F}{P_g} \nabla P_v$$

with $P_v$ being the vapor pressure

(7)

This last equation indicates the vapor flux vector $M_v$ is proportional to the gradient of the vapor pressure. To complete, the heat conduction in the porous medium is characterized by the Fourrier’s law:
\begin{equation}
q = -\lambda \nabla T
\end{equation}

with \(q\) is the thermal flux ([\text{J.m}^{-2}.\text{s}^{-1}]) and \(\lambda\) thermal conductivity ([\text{J.K}^{-1}.\text{m}^{-1}.\text{s}^{-1}]) . The energy conservation takes into account in one hand thermal sources and thermal flux and on the other hand the enthalpies evolution, i.e:

\[\sum_i h_i^m \Delta \Phi + \sum_i \text{div}(h_i^m M_i) + \text{div}(q) + Q' = \Theta \quad \text{with} \quad i = \text{liquid, dry air, vapour, dissolved gas} \quad (9)\]

In this equation \(h_i^m\) is the mass enthalpy of the phase \(i\), \(m_i\) represents mass content of phase \(i\), \(Q'\) is the phase change enthalpy and \(\Theta\) represents the thermal source terms.

Finally, by using the condition of perfect gases, the capillary pressure can be evaluated from the relative humidity with the Kelvin’s law

\[P_c = \frac{\rho_v RT}{M_i^v} \ln(Hr) \quad (10)\]

where \(R\), \(M_i^v\) are, respectively, the constant of perfect gases the molar mass of water and \(Hr\) is relative humidity.

**BACK ANALYSE OF IN SITU MEASUREMENTS**

A preliminary analysis of collected data from SACRE program is performed with objective to calibrate the model, for which a lot of data are necessary and often not known. In this paper most of hydro-mechanical properties of the “tuffeau blanc” stone are taken from laboratory results either from the work of Beck, 2006 ([2]) or from complementary laboratory test on thermal properties performed in the framework of SACRE program. The key parameters of the model are summarized in the Tab. (1). Note however that these data are not always in accord with those measured in laboratory. In fact some differences existent between the samples tested in laboratory and stones on the Chambord walls, not only because of natural heterogeneity of the stone but above all because some weathering is susceptible to exist on the surface of stone walls. From this point of view the differences on the properties between stones in situ and samples in laboratory may be interpreted as a signature of stone weathering.

The figure (1a) introduces the in-situ measurement of the temperature and relative humidity effectuated on the outside face of the Chambord’s wall. The red point in the figure present the sensor position which will measure the temperature and relative humidity at different points follow the thickness of the wall. Actually, these measurements were executed at the 0mm, 15mm, 30mm, 50mm and 250mm from the outside surface.
Figure 1: In-situ measurement of temperature and relative humidity (a); conceptual model, geometry mesh and boundary conditions of the section simulated (b)

In the figure (2) and (3) are presented the result determined by the sensor at different points on two similar boreholes (one in the south and the other on north surface of the Chambord) during two months from the 16th July to 14th September on 2009. For simplicity, firstly the daily average values of temperature or relative humidity are considered (one measure of temperature and humidity is taken each 15 minutes). These results show a tendency of the distribution of temperature and relative humidity in the bloc of the wall. We can observe that following the thickness of the wall the fluctuation of the temperature decrease. The maximum variations are situated on the surface (0mm) while the minimum is at the 250mm of depth. For the considered period the temperature fluctuates between 15°C and 28°C on the south face and it is of order 14°C to 26°C on the north. The fluctuation of relative humidity is more important on the south face (RH between 45% and 95 %) in comparison with the north surface (RH between 67% and 81%). However, in both case the relative humidity at 250mm of depth is almost constant ~74%. Some differences in the temperature amplitude and variations between two boreholes are also observed but they are relatively smaller as compared to the differences on RH.

Figure 2: Result of temperature and relative humidity of in-situ measurement at different depth
from the wall surface (south borehole).

Figure 3: Result of temperature and relative humidity of in-situ measurement at different depth from the wall surface (north borehole).

The objective of back analysis is to identify in-situ thermo-hydraulic parameters by trying to match, through coupled analyses based on the model already described, the measured data. The conceptual model of the wall with its geometry mesh and boundary conditions used in numerical simulations is shown in the figure (1b). Following the direction of the borehole the problem is merely 1D, since each point of the outside face has exactly the same temperature and RH history, and in the outside face of the wall, the temperature and RH are kept constant. It follows that it is sufficient to study only the variations along a horizontal line normal to two faces. For technical reasons this line is replaced by a thin layer of 5 cm in width and 40cm in length. The horizontal displacement is blocked on the left and right frontier of the model. At the inside face of the building which corresponds to the upper boundary we assume conditions of constant temperature ($T = 20^\circ C$) and relative humidity ($HR = 74\%$). Otherwise the outside face of the wall is in contact with the air for which the temperature and hygrometry changed as function of the environmental conditions. In Code_Aster concerning the conditions of the relative humidity, it is applied by using the capillary pressure determined from the Kelvin’s laws (Eq. 9). Moreover, the first data of temperature and humidity measured at different points of control are used as the initial state of the model ($t = t_0$).

**Table 1:** Properties of tuffeau used in calculations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus</td>
<td>$E = 1953(MPa)$</td>
</tr>
<tr>
<td>Poisson’s coefficient</td>
<td>$\nu = 0.19$</td>
</tr>
<tr>
<td>Mass density</td>
<td>$\rho = 1300(kg.m^{-3})$</td>
</tr>
<tr>
<td>Intrinsic permeability</td>
<td>$k_{in} = 2 \times 10^{-18}(m^2)$</td>
</tr>
<tr>
<td>Porosity</td>
<td>$\phi = 0.42$</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>$\lambda = 0.56(W.m^{-1}.K^{-1})$</td>
</tr>
<tr>
<td>Heat capacity</td>
<td>$C_p = 830(J.kg^{-1}.K^{-1})$</td>
</tr>
<tr>
<td>Biot’s coefficient</td>
<td>$b = 0.5$</td>
</tr>
<tr>
<td>Thermal expansion coefficient</td>
<td>$\alpha = 1.1 \times 10^{-5}(K^{-1})$</td>
</tr>
<tr>
<td>Isothermal sorption curve</td>
<td>$S_{iq}(P_c)$</td>
</tr>
<tr>
<td>Coupling parameters</td>
<td>$P_r = 0.013(MPa), \ n = 1.37$</td>
</tr>
</tbody>
</table>

The very first simulations were performed using the laboratory defined parameters. It is observed
that results of these simulations did not match the variations of temperature and RH inside the wall. In particular it was not possible using such a model to follow the variations of temperature and RH at 50mm and 250mm. This model, identified in what it follows as homogeneous model (because the wall was considered as a single homogeneous layer) was replaced by a two-layer model (identified as heterogeneous model now on). In this two-layer model it is supposed that the first layer (of 15mm of thickness beginning from the outside face of the wall) is weathered to some extent such that the hydraulic parameters (permeability, retention curve, ...) are modified as compared with the healthy stone.

As shown in the figure 4 the prediction of these two configurations (homogeneous, one layer model and heterogeneous, two-layer model) are qualitatively and quantitatively quite different. It seems that the degree of weathering of the stone on the surface plays a fundamental role on the distribution of temperatures and RH inside the wall.

A set of numerical calculations are effectuated to examine the role of some essential parameters to the predictive result of temperature and relative humidity variation inside of the wall. These simulations show the important influence of the thermal conductivity, heat capacity, intrinsic permeability, isothermal sorption curve on the final response. As previously indicated the response in terms of relative hygrometry will depend on the Fick diffusion and generalized Darcy advection. We try to fit the measured data by varying the effective conductivity. In fact the relative permeability is an ill-known function that needs to be fixed by this analysis. In order to fit the measured data the effective conductivity of liquid phase must be ten thousand times smaller than saturated conductivity parameter evaluated in the laboratory (see table 1 and reference [2]).

![Figure 4: Illustration of qualitative and quantitative differences of predictions obtained by one-layer (homogeneous) and two-layer (heterogeneous) model](image)

Not only the heterogeneous model fits better the measured data but also it seems to be closer to the
conceptual model of stone weathering. Observations in microscope and mineralogical analyses show the presence of stone alterations in the very first centimeters from the outside surface. From now on only results of heterogeneous model are considered.

In figures (5-8) the predictions of numerical model are presented against the measured data from both north-face and south-face boreholes. The same predictions are compared with measured data from north and south borehole.

The model seems able to follow quite well the day to day variations of temperatures. The relative discrepancies between measured and predicted values at depth 250mm would be explained, at least partially from the uncertainties on the geometry of the wall itself since the thickness of the walls in Chambord is not constant. The same could be said for the evolution of relative humidity in the south borehole when a good general agreement is obtained between numerical predictions and measured data. Despite the variations of the HR and temperature in the surface the fluctuations inside the wall are limited. Nevertheless the predictions of the model are closer to the measured data in north borehole than in south. This is particularly true for the north borehole. The discrepancies between predictions of the model and measured data on terms of temperature and relative humidity are greater in the case of south borehole. Since in numerical analyses the same parameters are used for both boreholes, these differences show also a different behavior of stone in the north and south borehole, which in turn might indicate differences on mechanisms and/or intensity of weathering.

![Temperature at 15mm from surface](image1)

![Temperature at 30mm from surface](image2)

![Temperature at 50mm from surface](image3)

![Temperature at 250mm from surface](image4)

**Figure 5:** Evolution of temperature at different depth from the wall surface (north borehole).
Figure 6: Evolution of RH at different depth from the wall surface (north borehole).

Figure 7: Evolution of temperature at different depth from the wall surface (south borehole).
IMPACT OF DAILY TEMPERATURE AND RELATIVE HUMIDITY ON STONE WEATHERING

After assessment of the key parameters, the model is used to perform some analyses of the walls taking into account the presence of the mortar in between stone blocks. The idea behind these analyses is to make an estimation of the role that could play in the stone weathering history the daily fluctuation of temperature and relative humidity. This is similar to work form Chau and Shao ([3]), however in this case realistic data are used for predictions. We considered a 80cm thick wall. The geometry and boundary conditions used in numerical analyses are shown in the figure 9 where also indicated some points where the variation of various quantities is followed during numerical simulation. A point INTF indicates a point in the interface stone-mortar, while points MR are points in the stone. In the outside surface the conditions applied on temperature and RH are those presented in the figure 10, obtained by a statistical analysis of time series on METEO FRANCE data near to Chambord castle.
The mechanical behavior of the structure is governed by effective stress as described in the beginning of this paper. An elastic behavior is adopted for stone and mortar. While the elastic properties of “tuffeau blanc” are known those of real mortar used in walls of Chambord are not known. In these simulations the parameters of a reconstituted mortar were used which may differ to some extents to the real values. The Biot’s coefficient for the mortar is taken equal to that of stone, while permeability is taken 10 times less that stone permeability. The thermal expansion coefficient of the mortar is twice that of stone. Generally speaking the parameters of the mortar are not so different from the parameters of stones.

The variations of effective stress in points inside the wall and in the interface stone-mortar are shown in the figure 11. It is shown that variations of temperature and relative humidity on the air are followed by variations of the effective stress in the wall. The amplitude of such variations is so big that at some given moments the apparition of the traction stress on the outside surface of the wall is observed. Note that the fluctuation of the stress is bigger in the wall than in inside of the stone and generally speaking on the outside surface the stress state is such that reaching of a traction stress state is easier than in inside surface. The fluctuations on climate conditions lead to a cyclic mechanical loading such that damage by fatigue becomes possible. It is to be noted that for simplicity, in our analyses only day to day variations of mean values have been taken into account leading to a smoother variation of temperature and RH. In reality these variations should be even higher since the variations between the maximums and minimums values of a given day is often bigger than the variation of mean values on two successive days.
The variation of effective stress in interface stone-mortar is not much different from that of in stone stress. Surely this result is highly dependent on the contrast of properties of stone and mortar and from that point of view one could argue that since the mortar used in calculation is not exactly the mortar used in Chambord wall construction, some differences are expected between these results and what really happened in situ. Nevertheless these results show that unless a very high contrast on properties of mortar and stone, the concentration of stress on the contact between stones and mortar would be moderate.

CONCLUSIONS

A thermo-hydro-mechanical model, adopted from poromechanics of unsaturated media is presented. The model is used to perform a back analyse of measured data in framework of SACRE research program. The comparison of simulated and measured data show that a skin effect due to stone weathering must be taken into account in the interpretation of these measures without which the global fitting of results is poor. Then, the model was used to assess the variations of effective stress due to thermo-hydro-mechanical coupling and day to day variation of temperature and relative humidity. These analyses show the apparition of stress fluctuations that at some moments become stress tractions. While these results show qualitatively that a fatigue like damage is possible, there is for the moment no quantitative characterisation of the fatigue on the stone in order to confirm/infirm this hypothesis. Likewise the role of the properties differences between stone and mortar is to be quantified, but the first estimation based on the properties of a
reconstituted mortar show no strong role of this interface on the stress concentration.

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