ANALYSIS OF AN ANCIENT ROMAN BRIDGE USING THE FINITE ELEMENT TECHNIQUE

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ABSTRACT
The rehabilitation and conservation has shown in recent years the need of reliable methods for assessing masonry arch bridges: it is important not only to maintain ancient structures in good conditions, but also, when necessary, to be able to estimate their safety factor as accurately as possible. The results of a 3-D finite element analysis of a stone masonry arch bridge in Aretha region, Patras, Greece are presented.

INTRODUCTION
Masonry arches were built since the beginning of the earliest civilization. The oldest reported arch bridge is the Arkadiko Bridge in Greece from about 1300 BC. Although the construction of arches was known by the ancient Greeks, the Roman engineers were the first to fully realize the potential of arches in building structures of large scale like bridges, aqueducts and public buildings (Terme di Caracalla et al [1]). Anyone approaching the study of masonry arch bridges will be amazed by the diversity of structural models and materials employed by the Roman engineers, in bridging a gap with an arch. The need for understanding the structural behavior of the Roman arch structures, together with the need of rehabilitation and conservation [2,3] has shown in recent years a lack of reliable methods for accessing masonry bridges. It is important not only to maintain ancient structures in good conditions, but also, when necessary to be able to estimate their safety factor as accurately as possible. The eventual distress of such structures can be linked to different causes such as an exceptional event or deterioration due to the effects of traffic and weathering. Since in stone masonry bridges dead loads are much larger from the live ones, the distress map can not change significantly its shape for the current employment of the structures, even if the traffic they are now required to carry is much heavier than that envisaged by the ancient designer. Thus, in several arch bridges crack patterns can be observed in the areas in which the load capacity of the structure is more involved. If the load history of the bridge excludes exceptional cases, such as the seismic load, in which the shape of the map is strongly different, the deterioration map coincides with the high stresses distribution.

The theoretical modeling of arch bridges considers two main different approaches: (1) a 2D one based on pre-elastic theories [4] and (ii) the 3D FEM approach. The former is a based on a classical limit analysis. As a result, of recent studies in structural mechanics, the latter has shown a great flexibility and a wide range of application fields [5].

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THE STRUCTURAL ANALYSIS

The case study is the Roman arch bridge in Patras, Greece, that is a stone bridge, nowadays in perfect conditions, for which the present analysis is applied. The great blocks by which it is build, arranged in a perfect ordered texture, were obtained from stone locally available, as it was usual in Roman Age. The circular arch has a span of 6 m, and a radius of 3 m. (Fig. 1).

![Figure 1: Roman arch bridge in Aretha, Patras, Greece.](image)

Together with the geometry, an accurate knowledge of the way in which it has been built, it is the first step towards the determination of the stress and deformation studies of the bridge. An accurate analysis of geometry and constituents of the bridge has been performed. A characteristic cross section of the bridge is shown in Fig. 2.

![Figure 2: Cross section of the bridge](image)

The bridge has been considered that is composed by one material with the characteristics reported in the following Table 1.

<table>
<thead>
<tr>
<th>Young s modulus (N/mm$^2$)</th>
<th>Strength (N/mm$^2$)</th>
<th>Poisson s ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5-6</td>
<td>0.2</td>
</tr>
</tbody>
</table>
The method of analysis is based on the idea that the composite is constructed explicitly from an initial material, stone, through a series of incremental additions, of stone and mortar. Due to the lack of experimental data the Poisson ratio was assumed equal to 0.2, although it has been shown that a variation in the Poisson ratio provides, sensible variation in the evaluation of the safety factor [6]. The inner core consists from cementium mass which is from mix pozolanic calcium with large pieces of inner materials (stones and bricks) (Fig 3).

Figure 3: General view of the bridge wall

NUMERICAL RESULTS AND DISCUSSION

For modeling the frame of Roman Bridge surface finite elements were used, of plane strain type, where the width of the structure was taken one meter along the length of the bridge. The characteristics of the construction materials used follow the EC 6 (structures from masonry). The applied loads, on the structure, were self weight, dead, live and seismic load that has characteristics corresponding to the region of Patras, Greece, according to Hellenic Antiseismic Regulations (HAR-2000) code. The results can be presented either in map isostatic stress ($\sigma_{xx}$, $\sigma_{yy}$, $\sigma_{xy}$, $\sigma_1$, $\sigma_2$, $\sigma_{\text{M}}$ normal, shear, principal and Von Misses) or in the form of principal modes for dynamic analysis. The results for each case are separately presented in each case of loading and for envelope values (negative and positive values). According to the results of the analysis and the values of design strength of masonry, that are computed with respect the methodology of EC 6, one observes that in general the stresses that developed in the structure are smaller than the design strengths. Exception occurs only locally, in small regions of the structure, where tensional stresses are developed and they are larger than the design strengths. These are not significant and practically disappear after appropriate redistributions of stresses due to the plasticity of the material and formation of small cracks in these regions, at the same time, the compression stresses does not exceed the 40% of the design strength.
Four type of loads were applied in the Roman bridge: (a) self weight, that is computed from the materials and the geometry of the bridge (b) dead load, due to the pavement of the bridge, a value of 5kN/m was assigned for this loading (c) live load of 33.3 kN/m was applied and (d) seismic load according to Hellenic Antiseismic Regulations (HAR 2000), or the EC 8. The support of the bridge was modeled by clamp in the foundation of the structure. The structure was analyzed using finite elements (the number of elements and nodes of the mesh are 1395 and 1536, respectively) (Fig 4). The results were plotted for the 1st mode of vibration, considering that the 1st displacement mode is the significant for the vibration analysis. Figure 5 indicates the local and global system of axes used in the present analysis of the bridge arches.

Figures 6a,b shows the distribution of stresses due to self weight as a function of the arch angle $\theta$. The normal stresses, $\sigma_{xx}$ along the local $x$-axis and it was found that it is equal to the principal stress $\sigma_2$ (see Fig. 5). According to the numerical results, the shear stresses $\tau_{xy}$ are insignificant (see Figs. 6a,b) which is expected for this type of loading. Figures 6a shows that the normal stress, $\sigma_{xx}$ or $\sigma_2$ has taken negative values i.e. compressive from the upper point of the arch ($\theta=0$) toward its base ($\theta=\pi/2$).
where is reaching its maximum values of 160kPa. In the case of dead loading no significant values of stresses appear. In the case of live loading, (Fig 6b) the behaviour of the normal stress $\sigma_{xx}$ is similar to the case of self weight, while the shear stress is insignificant. High values of stress are observed at the middle of arches and at the base of the bridge supports. The overall behavior of bridge under self weight is represented from the Von Mises component of stress that is shown in Figure 6b. The stress maps are indicated in Figs 7a,b.

In case of \textit{seismic excitation}, along the x-direction, according to HAR2000 the 1st mode of vibration is shown in Fig 8. Maximum stresses (380 kPa) are observed near the base of the bridge. According to HAR-2000 the spectrum acceleration is as follows:

$$R_d(T) = A \gamma \beta_d(T) q \eta \theta$$

where the parameters of the last equation are given in Table 2.

\textbf{Table 2: Characteristic seismic data for the region of Patras, Greece}

<table>
<thead>
<tr>
<th>$A$</th>
<th>$\gamma$</th>
<th>$\beta_d$</th>
<th>$q$</th>
<th>$n$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.24g (seismic region III)</td>
<td>0.85</td>
<td>2.5</td>
<td>1.5</td>
<td>0.90</td>
<td>1</td>
</tr>
<tr>
<td>(ground B)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\textbf{Figure 6a:} Normal, shear and Von Misses stress along the arch of the bridge ($\theta=0$ top of the arch, $\theta=\pi/2$ base of the arch) for \textit{self weight} loading conditions
**Figure 6b:** Normal, shear and Von Misses stress along the arch of the bridge ($\theta=0$ top of the arch, $\theta=\pi/2$ base of the arch) for the case of live loading.

**Figure 7a:** The $\sigma_{xx}$ (or $\sigma_2$) component of the stress for the case of self weight
**Figure 7b**: View of Von Misses (MPa) at the middle layer of width Cases: 1 (Self Weight)

**Figure 8**: View Von Misses (MPa) middle layer Cases: 5 (Seismic HAR-2000 Direction_X) Modes: 1; CQC
Table 3 indicates the comparison of the numerical results with the design strength values according to EC 6. In some cases the resulting tensile values are exceeding the design strength values but as it was mentioned before this over accumulation in local stresses is disappearing through the formation of small cracks.

Table 3: Tabularization of the results of analysis for the Roman Bridge

<table>
<thead>
<tr>
<th>Stress</th>
<th>$F_{w/(MPa)}$ Strength of masonry (MPa)</th>
<th>Loads</th>
<th>Loads</th>
<th>Description of Stresses</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{xx}^+$ (MPa)</td>
<td>0,04 MPa</td>
<td>0,01</td>
<td>0,00</td>
<td>0,01</td>
</tr>
<tr>
<td>$\sigma_{xx}^-$ (MPa)</td>
<td>1,67 MPa</td>
<td>-0,21</td>
<td>-0,03</td>
<td>-0,22</td>
</tr>
<tr>
<td>$\sigma_{yy}^+$ (MPa)</td>
<td>0,09 MPa</td>
<td>0,05</td>
<td>0,01</td>
<td>0,08</td>
</tr>
<tr>
<td>$\sigma_{yy}^-$ (MPa)</td>
<td>0,42 MPa</td>
<td>-0,06</td>
<td>-0,02</td>
<td>-0,11</td>
</tr>
<tr>
<td>$\sigma_{xy}^+$ (MPa)</td>
<td>0,43 MPa</td>
<td>0,04</td>
<td>0,01</td>
<td>0,06</td>
</tr>
<tr>
<td>$\sigma_{xy}^-$ (MPa)</td>
<td>0,43 MPa</td>
<td>-0,05</td>
<td>-0,01</td>
<td>-0,06</td>
</tr>
<tr>
<td>$\sigma_1$ (MPa)</td>
<td>-</td>
<td>0,05</td>
<td>0,01</td>
<td>0,09</td>
</tr>
<tr>
<td>$\sigma_2$ (MPa)</td>
<td>-</td>
<td>-0,20</td>
<td>-0,03</td>
<td>-0,20</td>
</tr>
<tr>
<td>$\sigma_w$ (von Misses)</td>
<td>-</td>
<td>0,18</td>
<td>0,03</td>
<td>0,19</td>
</tr>
</tbody>
</table>
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REFERENCES