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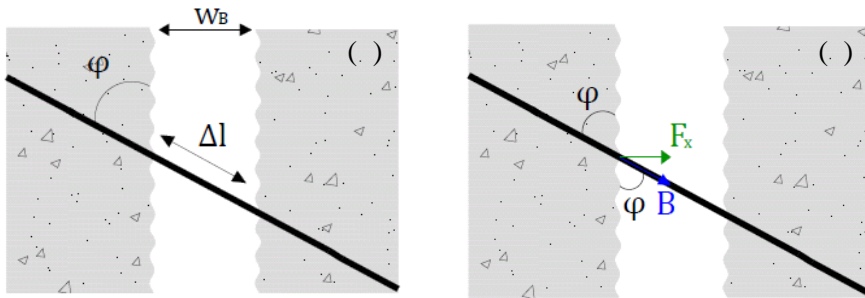
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$$F_{x,\varphi} = n \cdot \sin(\varphi) \cdot \Delta l$$

$$F_{x,tot} = \sum (n_{\varphi} \cdot F_{x,\varphi})$$



3: $w_B(\varphi)$ $F_x(\varphi), \mu$

$$\frac{D}{B} = \tan(\alpha) \quad (90^\circ)$$

$$\left(\frac{D}{B}\right)^3 + \left(\frac{D}{B}\right) = 1$$

6.1.2.2., (6.10)

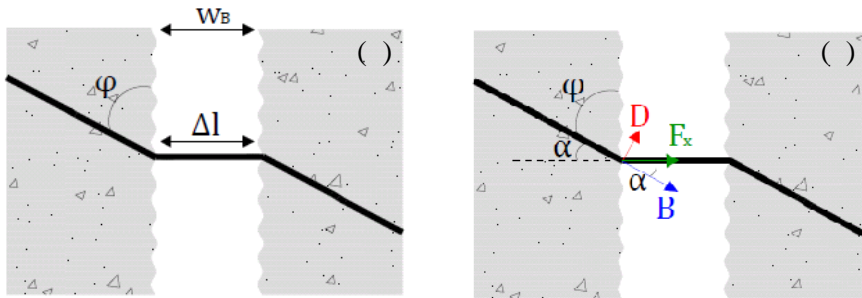
$$D = B \cdot \tan(\alpha) \quad (90^\circ)$$

$$D_{ud} = \frac{1.3 \cdot d_f^2}{\gamma_{Rd}} \sqrt{f_{cd} \cdot f_{yd}} \leq \frac{A_f \cdot f_{yd}}{\sqrt{3}}$$

(μ 1).

$$w_B = l \cdot (\sigma / E_s) \cdot I_1 \quad (4)$$

$$F_x = \sigma \cdot \cos(\alpha) \quad (4)$$



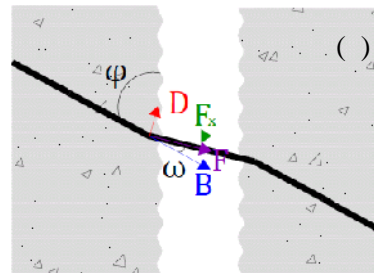
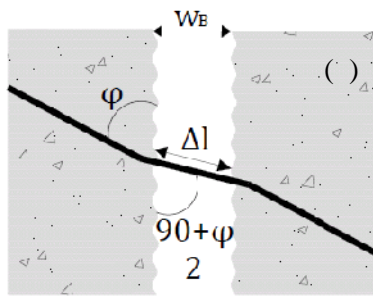
4: $w_B(\varphi)$ $F_x(\varphi), \mu$ 90°

3 _____ :
 $(90^\circ + \varphi)/2$.

$(90^\circ + \varphi)/2$.

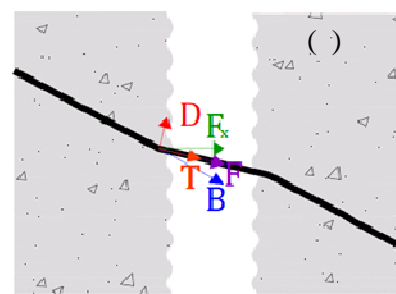
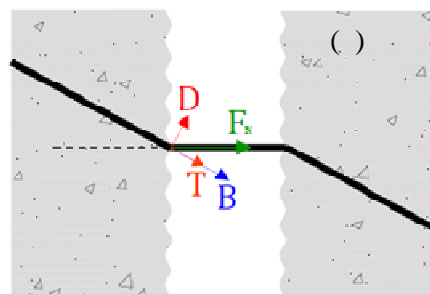
2 ,
 $D = B \cdot \sin \varphi$, $\varphi = (90^\circ - \alpha)/2$
 $(F/w)^{3/2} + (D/D_w)^{3/2} = 1 \Rightarrow (F \cos \alpha / w)^{3/2} + (B \sin \alpha / D_w)^{3/2} = 1$.

$w_B = l \cdot \sin \alpha = (F/w) / E_s \cdot l_1 \cdot \sin \alpha$ (.5)
 $F_x = F \cdot \cos \alpha = F \cdot \cos^2 \alpha$ (.5)



.5: w_B () F_x () $(90^\circ + \varphi)/2$

$D = \mu \cdot D$
 $\mu = \mu \cdot D$ (.6).



.6: 90° () $(90^\circ + \varphi)/2$ ()

$$\left(\frac{B+T}{B_{ud}} \right)^{3/2} + \left(\frac{D}{D_{ud}} \right)^{3/2} = 1$$

$$F_x = (\dots) \cdot \cos [(90^\circ + \dots) / 2]$$

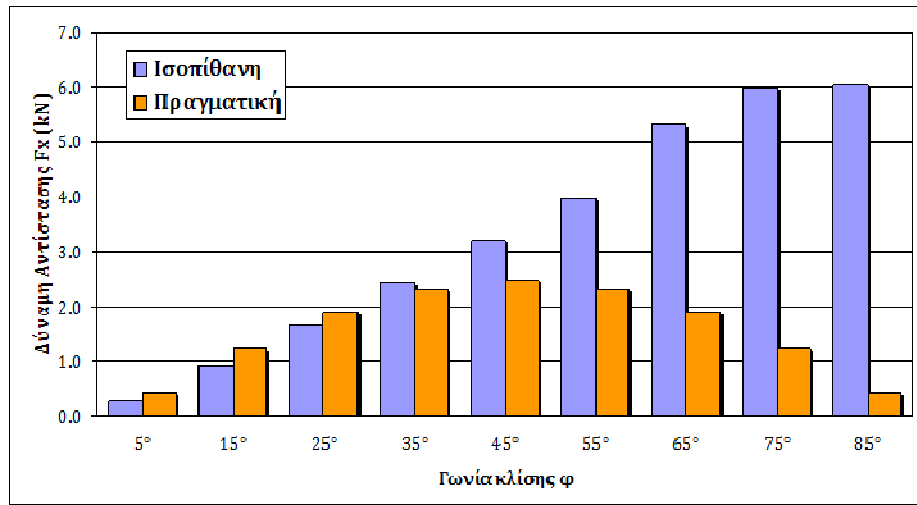
$$F = B \cdot \cos \omega + \dots$$

$$\left(\frac{B \cdot \cos \omega + T}{B_{ud}} \right)^{3/2} + \left(\frac{D}{D_{ud}} \right)^{3/2} = 1$$

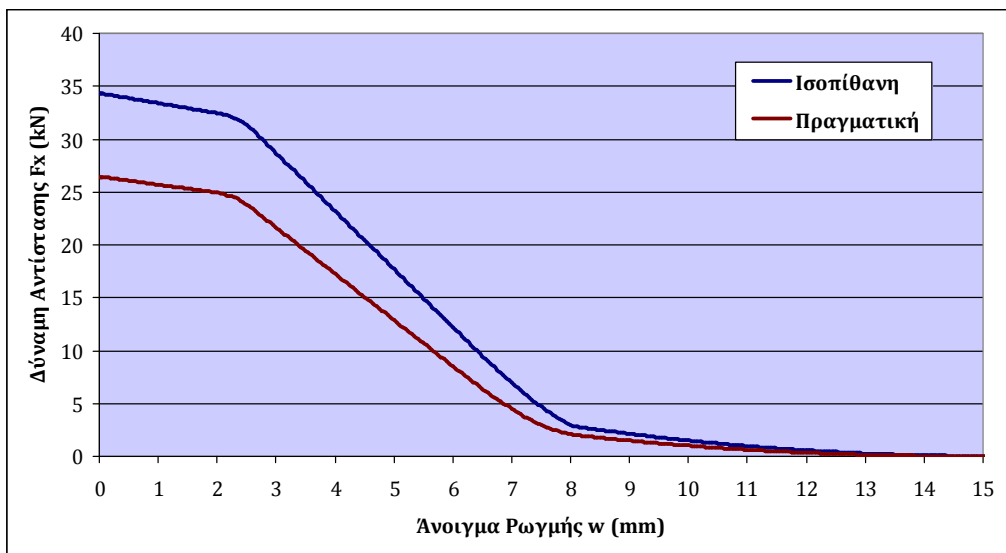
Η σχέση $\left(\frac{B+T}{B_{ud}} \right)^{3/2} + \left(\frac{D}{D_{ud}} \right)^{3/2} = 1$ προκύπτει από την ισορροπία των δυνάμεων στο σημείο της άρθρωσης. Η δύναμη F_x είναι η οριζόντια συνιστώσα της δύναμης F . Η γωνία ω είναι η γωνία που σχηματίζει η δύναμη F με την οριζόντια. Η σχέση $F = B \cdot \cos \omega + \dots$ προκύπτει από την ισορροπία των δυνάμεων στο σημείο της άρθρωσης. Η δύναμη F_x είναι η οριζόντια συνιστώσα της δύναμης F . Η γωνία ω είναι η γωνία που σχηματίζει η δύναμη F με την οριζόντια. Η σχέση $\left(\frac{B \cdot \cos \omega + T}{B_{ud}} \right)^{3/2} + \left(\frac{D}{D_{ud}} \right)^{3/2} = 1$ προκύπτει από την ισορροπία των δυνάμεων στο σημείο της άρθρωσης. Η δύναμη F_x είναι η οριζόντια συνιστώσα της δύναμης F . Η γωνία ω είναι η γωνία που σχηματίζει η δύναμη F με την οριζόντια.

$$n_\varphi = P(\varphi) \cdot \frac{V_{\lambda\omega\varphi}}{V_f} \cdot \{ \dots \}$$

Η σχέση $n_\varphi = P(\varphi) \cdot \frac{V_{\lambda\omega\varphi}}{V_f} \cdot \{ \dots \}$ προκύπτει από την ισορροπία των δυνάμεων στο σημείο της άρθρωσης. Η δύναμη F_x είναι η οριζόντια συνιστώσα της δύναμης F . Η γωνία ω είναι η γωνία που σχηματίζει η δύναμη F με την οριζόντια. Η σχέση $\left(\frac{B \cdot \cos \omega + T}{B_{ud}} \right)^{3/2} + \left(\frac{D}{D_{ud}} \right)^{3/2} = 1$ προκύπτει από την ισορροπία των δυνάμεων στο σημείο της άρθρωσης. Η δύναμη F_x είναι η οριζόντια συνιστώσα της δύναμης F . Η γωνία ω είναι η γωνία που σχηματίζει η δύναμη F με την οριζόντια.



7.7 : F_x (kN) vs ϕ (degrees). The chart compares the axial force F_x for two concrete models: Ισοπίθκη (Isotropic, blue bars) and Πραγματική (Real, orange bars). The x-axis represents the angle of inclination ϕ from 5° to 85°. The y-axis represents the axial force F_x in kN, ranging from 0.0 to 7.0. The Ισοπίθκη model shows a general increase in force as the angle increases, while the Πραγματική model shows a more complex, non-monotonic behavior, peaking around 25°-35° and then decreasing significantly at higher angles.



7.8 : $F_{x,tot}$ (kN) vs w (mm). The chart compares the total axial force $F_{x,tot}$ for two concrete models: Ισοπίθκη (Isotropic, blue line) and Πραγματική (Real, red line). The x-axis represents the opening width w in mm, ranging from 0 to 15. The y-axis represents the total axial force $F_{x,tot}$ in kN, ranging from 0 to 40. Both models show a decrease in force as the opening width increases, with the Ισοπίθκη model consistently showing higher force than the Πραγματική model. The force decreases significantly after $w = 3$ mm.

$$F_{x,tot,90^\circ} = \frac{F_{x,tot}}{\sqrt{2}} = \frac{75.557}{\sqrt{2}} = 53.557 \text{ kN}$$

$$f_{x,tot,90^\circ} = \frac{f_{x,tot}}{\sqrt{2}} = \frac{0.3}{\sqrt{2}} = 0.214 \text{ MPa}$$

$$\bar{B} = 0.2443 \text{ kN}$$

$$n = 219.23$$

$$F_{x,tot,90^\circ} = 219.23 \cdot 0.2443 = 53.557 \text{ kN}$$

$$2 : \quad \mu \quad F_{x,tot} \quad f_{x,tot} \quad \mu$$

| μ | | μ | | μ | |
|-----------|---|-------------------------|--------------------------|-------------------------|--------------------------|
| | | F _{x,tot} (kN) | f _{x,tot} (MPa) | F _{x,tot} (kN) | f _{x,tot} (MPa) |
| | | 26.778 | 0.298 | 17.898 | 0.199 |
| 90° | | 22.812 | 0.253 | 14.512 | 0.161 |
| | μ | 24.513 | 0.272 | 15.760 | 0.175 |
| | μ | 33.775 | 0.375 | 24.876 | 0.276 |
| (90°+)/2 | | 27.823 | 0.309 | 19.454 | 0.216 |
| | μ | 29.815 | 0.331 | 21.233 | 0.236 |
| | μ | 34.314 | 0.381 | 26.416 | 0.294 |

3 :

| μ | | (kN) | |
|-----------|---|--------------------------|--------|
| | | 0.2443 | 5°–85° |
| 90° | | 0.2443 | 85° |
| | μ | 0.2539 | 85° |
| | μ | 0.4418 = B _{ud} | 5°–15° |
| (90°+)/2 | | 0.2441 | 85° |
| | μ | 0.2566 | 75° |
| | μ | 0.2678 | 45° |

4 : μ F_{x,tot} μ F_{x,tot,90°},
μ μ

| μ | | μ | | μ | |
|-----------|-----|-------------------------|--|-------------------------|--|
| μ | | F _{x,tot} (kN) | F _{x,tot} /F _{x,tot,90°} | F _{x,tot} (kN) | F _{x,tot} /F _{x,tot,90°} |
| | | 26.778 | 50.0 % | 17.898 | 33.4 % |
| 90° | | 22.812 | 42.6 % | 14.512 | 27.1 % |
| | μ . | 24.513 | 45.8 % | 15.760 | 29.4 % |
| | μ . | 33.775 | 63.1 % | 24.876 | 46.4 % |
| (90°+)/2 | | 27.823 | 52.0 % | 19.454 | 36.3 % |
| | μ . | 29.815 | 55.7 % | 21.233 | 39.6 % |
| | μ . | 34.314 | 64.1 % | 26.416 | 49.3 % |

μ (90°+)/2 μ ,
μ μ (5, 6),
μ μ ,
:
– μ
– μ

5 : μ F_{x,tot} , μ (90°+)/2

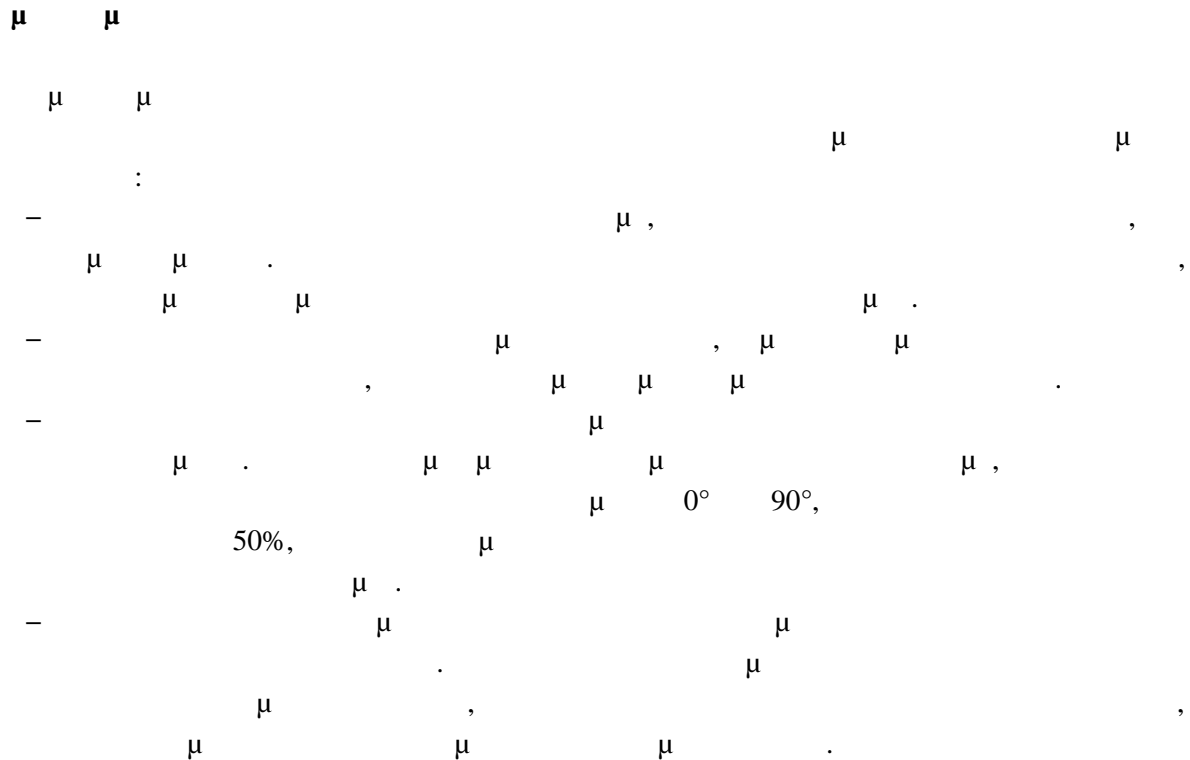
| | | F _{x,tot} (kN) | | | |
|-------------------|-----------------------|-------------------------|--------|--------|--------|
| kg m ³ | | μ | | μ | |
| μ | | μ | μ | μ | μ |
| 0.25 % | 20 kg/m ³ | 15.021 | 10.685 | 17.355 | 13.361 |
| 0.50 % | 40 kg/m ³ | 29.815 | 21.233 | 34.314 | 26.416 |
| 0.75 % | 60 kg/m ³ | 44.459 | 31.690 | 51.027 | 39.283 |
| 1.00 % | 80 kg/m ³ | 59.986 | 42.076 | 67.549 | 52.003 |
| 1.25 % | 100 kg/m ³ | 73.419 | 52.406 | 83.911 | 64.598 |

| | | | | | |
|--------|-----------------------|--------|--------|---------|--------|
| 1.50 % | 120 kg/m ³ | 87.768 | 62.685 | 100.132 | 77.086 |
|--------|-----------------------|--------|--------|---------|--------|

$\delta :$ μ $F_{x,tot}$ μ μ $l/d,$
 μ $d_f=0.75 \text{ mm}, \mu$ $(90^\circ+)/2$

| μ l/d | μ $F_{x,tot} \text{ (kN)}$ | | | |
|----------------|--------------------------------|--------|---------|--------|
| | μ | | μ | |
| | μ | μ | μ | μ |
| 40 | 29.815 | 21.233 | 34.314 | 26.416 |
| 50 | 48.224 | 34.192 | 56.408 | 43.426 |
| 60 | 61.590 | 43.522 | 73.260 | 56.399 |
| 70 | 72.777 | 51.296 | 88.203 | 67.902 |
| 80 | 82.372 | 57.933 | 101.729 | 78.316 |
| 90 | 90.811 | 63.717 | 114.201 | 87.917 |
| 100 | 98.149 | 68.788 | 125.818 | 96.839 |

μ l/d , μ $F_{x,tot}, \mu$ μ
 μ μ μ μ



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