RECENT ADVANCES IN OPTIMUM DESIGN OF STEEL LATTICED DOMES

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1. SUMMARY
In this work, we focus on the recently developed optimum design procedures of steel latticed domes, which are various metaheuristic search techniques that strongly employ randomized decisions while searching for solutions to structural optimization problems. These novel and innovative techniques make use of ideas inspired from the nature and do not suffer the discrepancies of mathematical programming based optimum design methods. The basic idea behind these is to simulate the natural phenomenon, they don’t require the gradient information of the objective function and constraints and they use probabilistic transition rules not deterministic ones. Eight characteristic methods are discussed, namely: simulated annealing, genetic algorithms, evolution strategies, particle swarm optimizer, tabu search, ant colony optimization, harmony search and charged system search. Their main features are described, while the convergence rates and reliabilities in attaining the optimum solution for the structures are also compared via a benchmark design example.

2. INTRODUCTION
Steel domes are elegant structures, which provide cost-effective solutions for covering large areas without intermediate supports. In fact, a dome is a structural system that consists of one or more layers of elements that are arched in all directions. The surface of a dome may be a part of a single surface (usually of revolution, such as a sphere or a
paraboloid), or it may consist of a patchwork of different surfaces. The most commonly used basic single layer latticed dome configurations are: ribbed, Schwedler, lamella, diamatic, two- or three-way grid, geodesic and network. In particular, the possibility of constructing such structures, with steel as the main material, in considerably low costs has increased in popularity. Nevertheless, the variety in steel profiles with tube-shaped cross-sections as well as the multitude of industrial patent joints in conjunction with the dimension and design requirements leads to a very complex problem, when optimization is concerned. Evidently, this optimum design under prescribed loading conditions requires the finding of optimal sections for elements (size), optimal height for the crown (geometry) and optimum number of elements (topology), combined with best performance for local and global buckling as well as strength and serviceability, given their strongly nonlinear response. Hence, if classical deterministic optimization techniques, governed by design variables of continuous type, are used, they will generate solutions not correctly matching available cross-sectional properties of a ready steel profile list. In addition to the above, when the solution space has irregular peaks, these gradient based approaches fail to produce fair and accurate gradient information. These are the reasons why non-classical, stochastic optimization schemes denoted as metaheuristics and governed by probabilistic transition rules, have recently found their way to the problem of optimum design of steel latticed domes.

In view of the above, the present work briefly reflects the recent advances and trends in the optimum design of steel latticed domes, accounting for eight characteristic metaheuristic techniques (listed in the Abstract) that have been applied to the foregoing problem. After presenting general aspects of structural optimization and outlining the general features of the techniques dealt with, a short comparison of their effectiveness, convergence rates and reliability via a benchmark example are given.

3. BACKGROUND - GENERAL ASPECTS OF STRUCTURAL OPTIMIZATION

Structural design may be generally described as the ability to create innovative and highly appropriate structural solutions to both familiar and new problems, balancing efficiency, economy, formal elegance and utility [1]. Embodying this definition, lead to the development of numerous structural optimization procedures, which have drawn huge attention and interest over the past four decades, mainly as far as optimality criteria and mathematical programming are concerned. For engineering structures, optimum design can be thought of as an improvement of a proposed one resulting in the best overall properties for minimum cost [2]. In this manner, the whole problem may be classified as [3]:

- Sizing optimization, in which one seeks optimal sections for elements, with the geometry and the topology of the structure remaining unchanged,
- Geometry optimization, which determines the optimum location of the joints in the structure in addition to the size of members, and
- Topology optimization, that involves finding the number of members in the structure and their corresponding connectivity, i.e. the way in which these members are connected to each other.

The most general mathematical form of a structural optimization (SO) problem is given in eq. (1), in which $i$, $j$ and $k$ are indexes, with values depending on the individuality of the problem at hand:
minimize $f_k(x_i, y_j)$ with respect to $x_i, y_j$

subject to

behavioral constraints on $x_i, y_j$

design contraints on $x_i, y_j$
equilibrium and compatibility constraints
local and global stability constraints

where $f_k$ are the objective functions, while $x_i$ and $y_j$ are the sets of the design and state variables respectively.

Diverse traditional deterministic methods are capable of obtaining in a cost-effective manner a global optimal solution in problem models with certain particularities, but with a limited application range. This shortcoming, very often encountered in structural optimization, can be coped however via the use of metaheuristic techniques, conceived as intelligent self-learning algorithms stemming from the study and mimicking of processes and behaviors arising in nature. A general classification of the aforementioned methods is given in Fig.1, based on their operational procedure.

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**Fig. 1 General classification of metaheuristics based on their operational procedure**

### 4. SHORT DESCRIPTION OF METAHEURISTIC TECHNIQUES

The contents of the present section briefly reflect the underlying features of the metaheuristic methods discussed herein. For more detailed information one may refer to the relevant literature, as for instance the book by Yang [4].

#### 4.1 Simulated Annealing (SA)

This method locates a good approximation to the global optimum of a given function in a large mainly discrete search space. It is more effective than exhaustive enumeration (classical method), provided that the goal is merely to find an acceptable good solution in a
fixed amount of time, rather than the best possible solution. Both are attributes of the material that depend on its thermodynamic free energy. Heating and cooling the material affects both the temperature and the thermodynamic free energy. While the same amount of cooling brings the same amount of decrease in temperature it will bring a bigger or smaller decrease in the thermodynamic free energy depending on the rate that it occurs, with a slower rate producing a bigger decrease. This notion of slow cooling in implemented in the SA algorithm as a slow decrease in the probability of accepting worse solutions as it explores the solution space. Accepting worse solutions is a fundamental property of metaheuristics because it allows for a more extensive search for the optimal solution.

4.2 Genetic Algorithm (GA)
Genetic algorithms [5] are stochastic search techniques widely used in obtaining the solution of complex engineering optimization problems. Their name is borrowed from natural genetics because they are based on the simulation of natural genetics and survival of the fittest. In a GA a population of candidate solutions to an optimization problem is evolved toward better solutions. Each candidate solution has a set of properties which can be mutated and altered. The evolution usually starts from a population of randomly generated individuals, and is an iterative process, with the population in each iteration called a generation. In each generation, the fitness of every individual in the population is evaluated. The more fit individuals are stochastically selected from the current population, and each individual's genome is modified to form a new generation. The new generation of candidate solutions is then used in the next iteration of the algorithm.

4.3 Evolution Strategy (ES)
Based on ideas of adaptation and evolution, this strategy belongs to the general class of artificial evolution methodologies and uses natural problem-dependent representations, and primarily mutation and selection, as search operators. In common with evolutionary algorithms, the operators are applied in a loop. An iteration of the loop is called a generation. The sequence of generations is continued until a termination criterion is met.

4.4 Particle Swarm Optimization (PSO)
It is a computational method that optimizes a problem by iteratively trying to improve a candidate solution with regard to a given measure of quality. PSO optimizes a problem by having a population of candidate solutions, and moving these particles around in the search space according to simple mathematical formulae over the particle's position and velocity. PSO makes few or no assumptions about the problem being optimized and can search very large spaces of candidate solutions.

4.5 Tabu Search (TS)
This method uses a local or neighbourhood search procedure to iteratively move from one potential solution $x$ to an improved solution $x'$ in the vicinity of $x$, until some stopping criterion is satisfied. The solutions admitted to the new neighbourhood $N^*(x)$ are determined through the use of memory structures. Using these, the search progresses by iteratively moving from the current solution $x$ to an improved solution $x'$ in $N^*(x)$. These memory structures form what is known as the tabu list, a set of rules and banned solutions used to filter which solutions will be admitted to the neighborhood $N^*(x)$ to be explored by the search. More commonly, a tabu list consists of solutions that have changed by the process of moving from one solution to another. It is convenient, for ease of description, to understand a “solution” to be coded and represented by such attributes.
4.6 Ant Colony Optimization Algorithm (ACO)
It is a probabilistic technique for solving computational problems which can be reduced to finding good paths through graphs. In the natural world, ants (initially) wander randomly, and upon finding food return to their colony while laying down pheromone trails. If other ants find such a path, they are likely not to keep travelling at random, but to instead follow the trail, returning and reinforcing it if they eventually find food. The idea of the ant colony algorithm is to mimic this behavior with "simulated ants" walking around the graph representing the problem to solve.

4.7 Harmony Search (HS)
It is a relatively new population-based algorithm which has obtained excellent results in the field of combinatorial optimization. It mimics the behavior of a music orchestra when aiming at composing the most harmonious melody, in the sense of aesthetic standards. Just like the musician improves the melody as time evolves, the HS algorithm progressively enhances the fitness of the solution vector in an iterative fashion. For a state-of-the-art, extensive review and specific information about the foregoing method one may visit the newly developed website: www.harmonysearch.info.

4.8 Charged System Search (CSS)
This quite novel algorithm [6] is inspired by the Coulomb law of Electrostatics and the laws of motion from Newtonian Mechanics. It contains a number of agents called charged particles (CPs) that attract each other by a force dependent to their separation distance. It considers three essential concepts, self-adaptation, cooperation and competition and has been proven quite efficient and outperforming evolutionary algorithms.

5. APPLICATIONS ON OPTIMUM DESIGN OF STEEL LATTICED DOMES
In recent literature, one may find some individual applications of the eight methods considered for the optimum design of steel domes, as for instance GA for nonlinear braced and latticed domes [7-9], HS for network and latticed domes [10,11] and CSS for geodesic domes [2]. Only a limited number of studies however offer a comparison of these methods, and a characteristic one is the work by Hasancebi et al. [12]. In the present work we adopted the benchmark example on the optimum of the geodesic dome used in [12], and performed once again optimization using also CSS, which was not included in the original comparative study. The objective function (to be minimized) was the weight of the structure, and the variables and constraints included structural behavior (strength and stability), performance limitations (serviceability) according to ASD-AISC specifications (Allowable Stress Design, 9th edition, 1989), and selection of members sections from standard CHS tabulated ones. The plan, elevation and 3D view of the geodesic dome used is shown in Fig. 3. It has a base diameter of 20m, a total height of 4m and consists of 51 pinned-joints and 130 members grouped in 8 independent size design variables.

The design loads and combinations were adopted from ASCE 7 – 98 (minimum design loads for buildings and other structures) and included dead loads, snow and wind actions, temperature changes and earthquake actions. The material properties of the steel used were \( E=203893.6 \) MPa and \( f_y = 253.1 \) MPa. It should be noted that the algorithm for each metaheuristic was selected on the basis of generality and reported performance, and mainly from procedures embedded in modern mathematical software. The minimum weight design of the dome was sought by conducting three independent runs with each of the eight
techniques due to their stochastic natures, and 10 structural analyses were performed in each run (for giving equal opportunity to each scheme to grasp the global minimum). The results are given in Table 1.

![Fig. 2 3D view (a), side view (b) and elevation (c) of the 130-member geodesic dome](image)

<table>
<thead>
<tr>
<th>Runs</th>
<th>SA</th>
<th>ES</th>
<th>PSO</th>
<th>HS</th>
<th>TS</th>
<th>ACO</th>
<th>GA</th>
<th>CSS</th>
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<td>5.44</td>
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<td>5.61</td>
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<tr>
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<td>5.46</td>
<td>5.50</td>
<td>5.73</td>
<td>5.44</td>
</tr>
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</table>

*Tab. 1: Minimum weight designs in lbs/1000*

It is observed that SA, ES, PSO and CSS located the same solution in each of the three runs, a fact that evinces high convergence reliabilities in optimum design. On the other hand, HS, TS and ACO identified the optimum solution twice out of three runs, indicating that these are more susceptible to performance variations, while GA exhibited a rather substandard performance, since it identified the optimum only once. These qualitative ascertainments can also be perceived in the design history graph of Fig. 3.

![Fig. 3 Design history graph of the benchmark example](image)
6. CONCLUSIONS

All the techniques dealt with in this study managed to find the optimum solution in at least one of the three independent runs performed. ES, SA, PSO and CSS are more suitable as far as convergence and reliability are concerned, while TS, ACO and HS methods exhibited significant performance variations, ought to their more pronounced stochastic nature. GA did not show expected satisfactory performance, but this might depend on the nature of the optimization problem itself.

Most likely, it seems that it relies on the designer to choose the metaheuristic to be engaged into the optimization procedure at hand. Far more comparison studies must be performed in order to gain a robust insight of their overall applicability and performance, at least for steel latticed domes or related spatial structures.

7. REFERENCES

ΠΕΡΙΛΗΨΗ

Εστίαζουμε στις πρόσφατες αναπτυχθείσες διαδικασίες βέλτιστου σχεδιασμού χαλύβδινων θόλων, ήτοι σε διάφορες μετα-ευρετικές τεχνικές, που χρησιμοποιούν τυχαιοποιημένες αποφάσεις κατά την αναζήτηση βέλτιστων λύσεων. Αυτές οι μη παραδοσιακές στοχαστικές καινοτόμες μέθοδοι κάνουν χρήση ιδεών εμπνευσμένων από τη φύση και υπερτερούν των μεθόδων με βάση τον μαθηματικό προγραμματισμό. Προσομοιώνουν το σχετικό φυσικό φαινόμενο και δεν απαιτούν πληροφορίες κλίσης ούτε για την αντικειμενική συνάρτηση ούτε για τους περιορισμούς, κάνοντας χρήση πιθανολογικών και όχι αιτιοκρατικών κανόνων μετάβασης. Συζητούνται οι ακόλουθες εκτός τέτοιες μέθοδοι: προσομοιωμένης ανάπτυξης, γενετικών αλγορίθμων, στρατηγικών εξέλιξης, βέλτιστου σμήνους σωματιδίων, αναζήτησης απαγόρευσης, βέλτιστης αποικίας μυρμηγκιών, αναζήτησης αρμονίας και αναζήτησης ηλεκτρικά φορτισμένου συστήματος. Περιγράφονται τα κύρια χαρακτηριστικά τους και μέσω ενδεικτικού παραδείγματος σχεδιασμού ενός χαλύβδινου γεωδαιτικού θόλου συγκρίνονται οι τιμές σύγκλισης και η αξιοπιστία τους.